Generating solutions of "impossible-to-solve" problems and simulating "impossible-to-simulate" models

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Planting !

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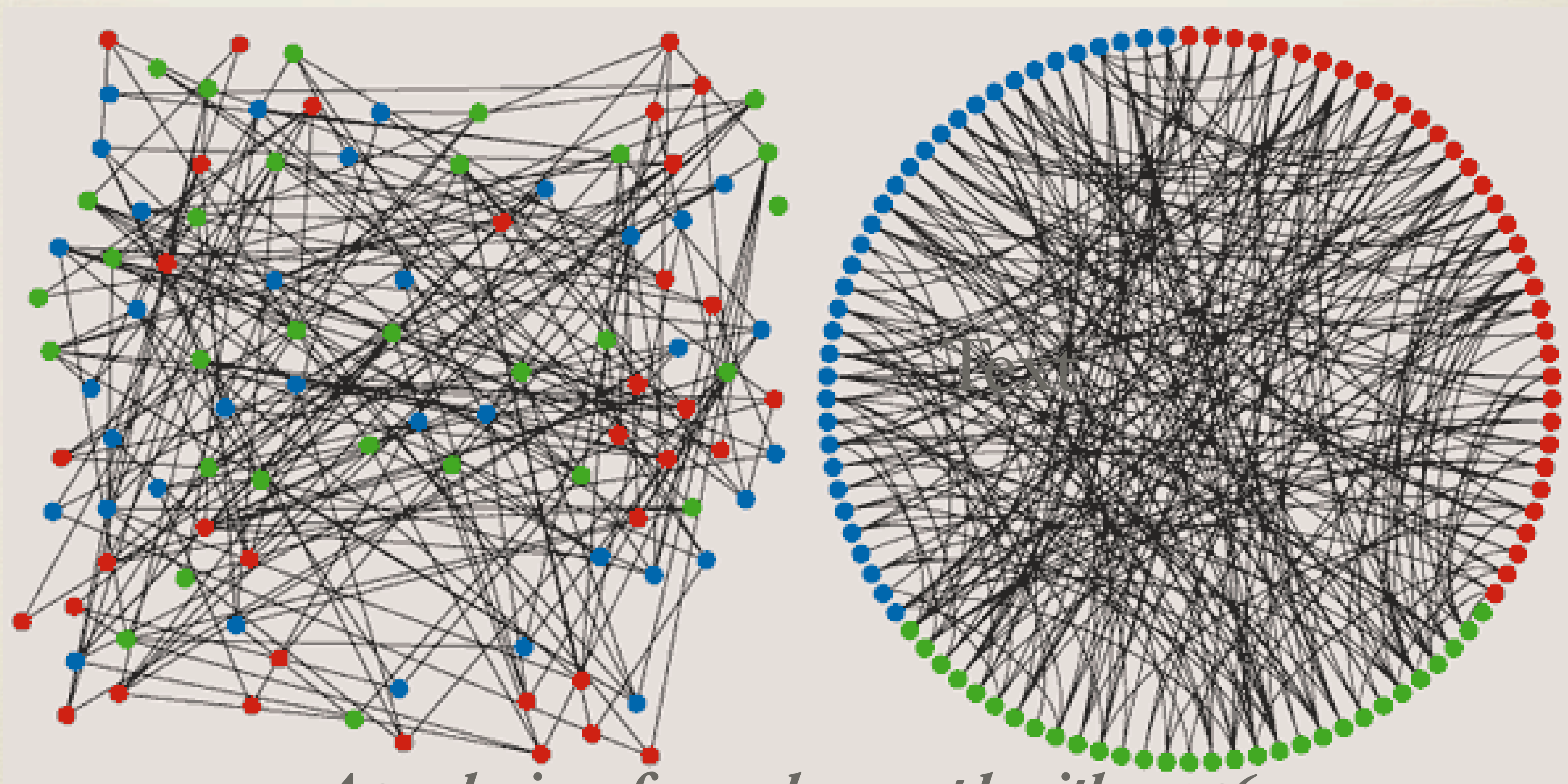
Impossible-to-solve problems

Some optimization problems such as COL and SAT are almost impossible to solve!

ex: Hard Instances of random graph coloring



A 3-coloring of a random graph with c=4.36

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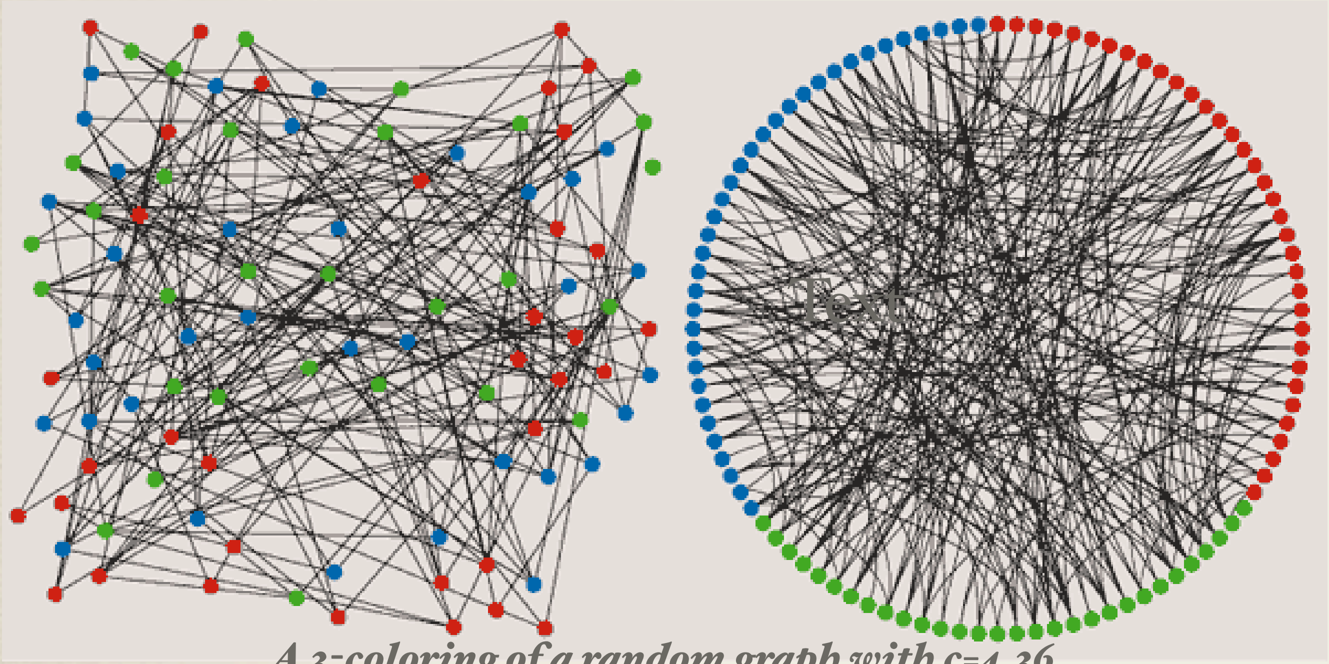

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D. Achlioptas et al. Nature 2005

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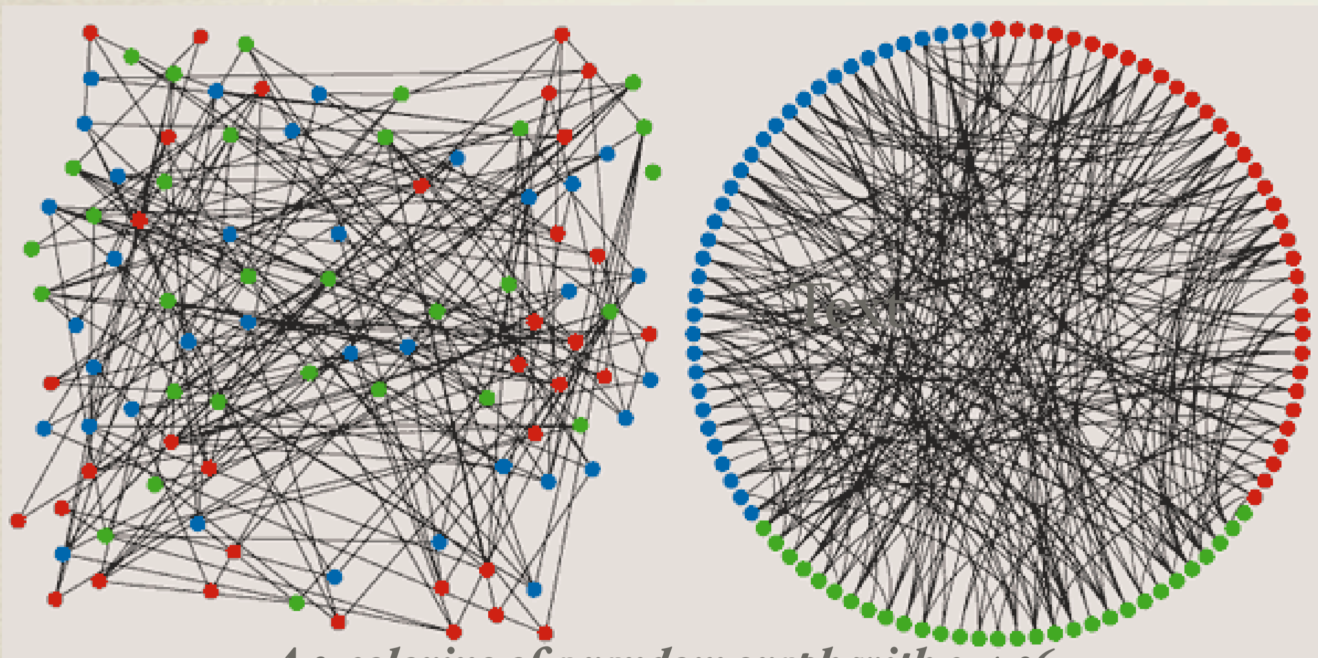

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- Consider q color (with q large enough) and a large random graph of average degree c
- W.h.p this graph is colorable if c<2q log q
- However, no algorithm is able to do so efficiently (polynomial) for c> q log q !

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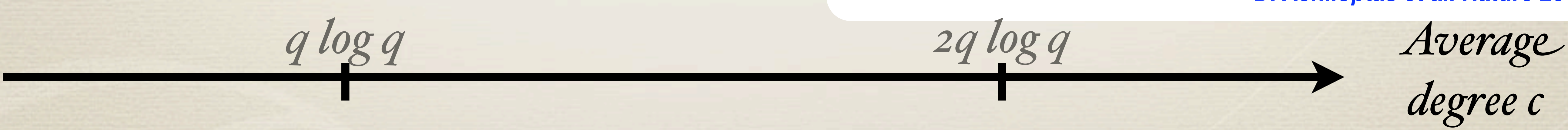

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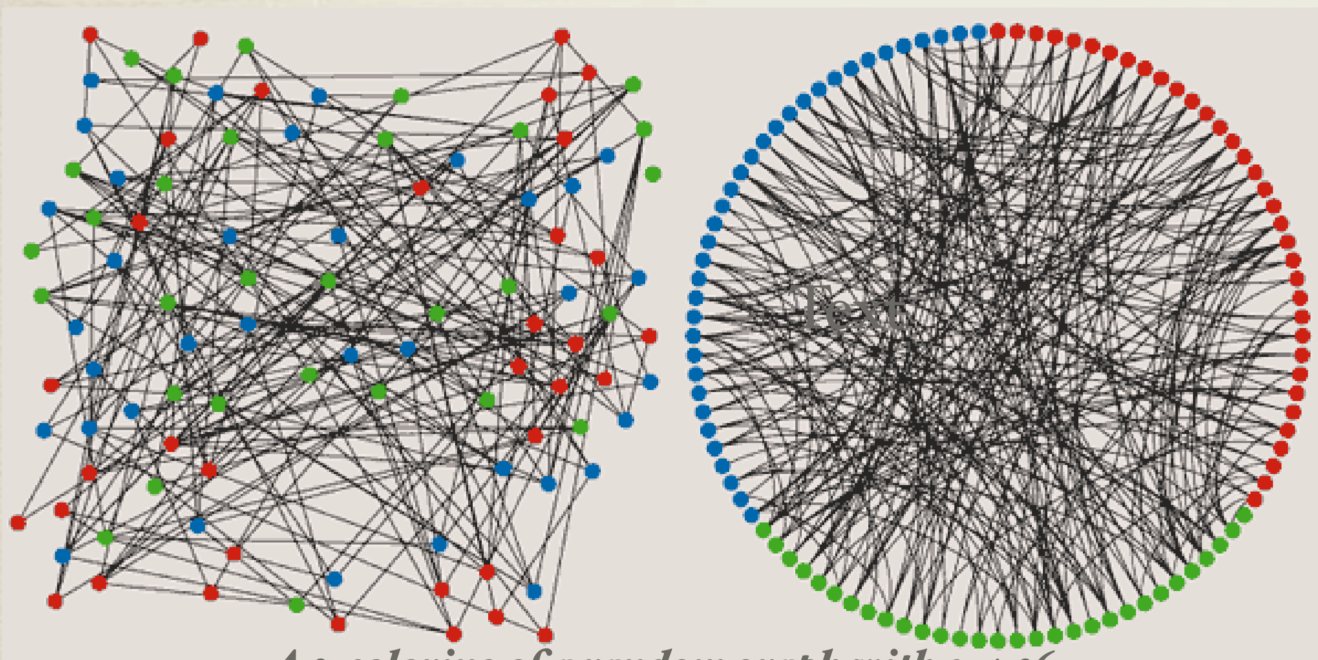

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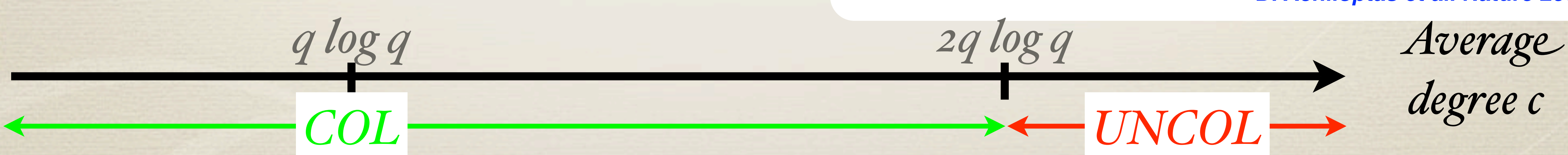

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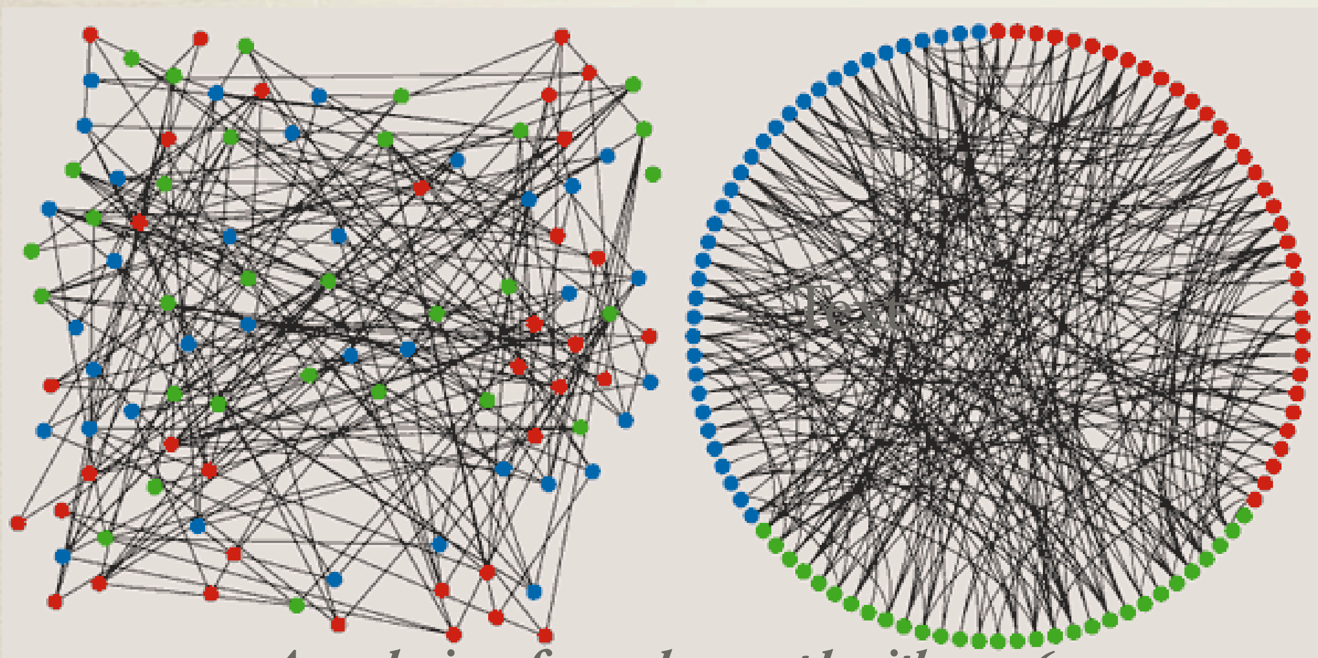

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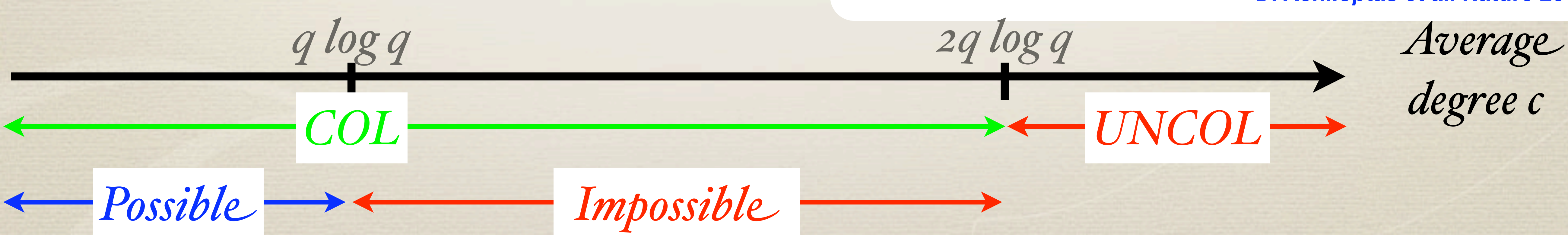

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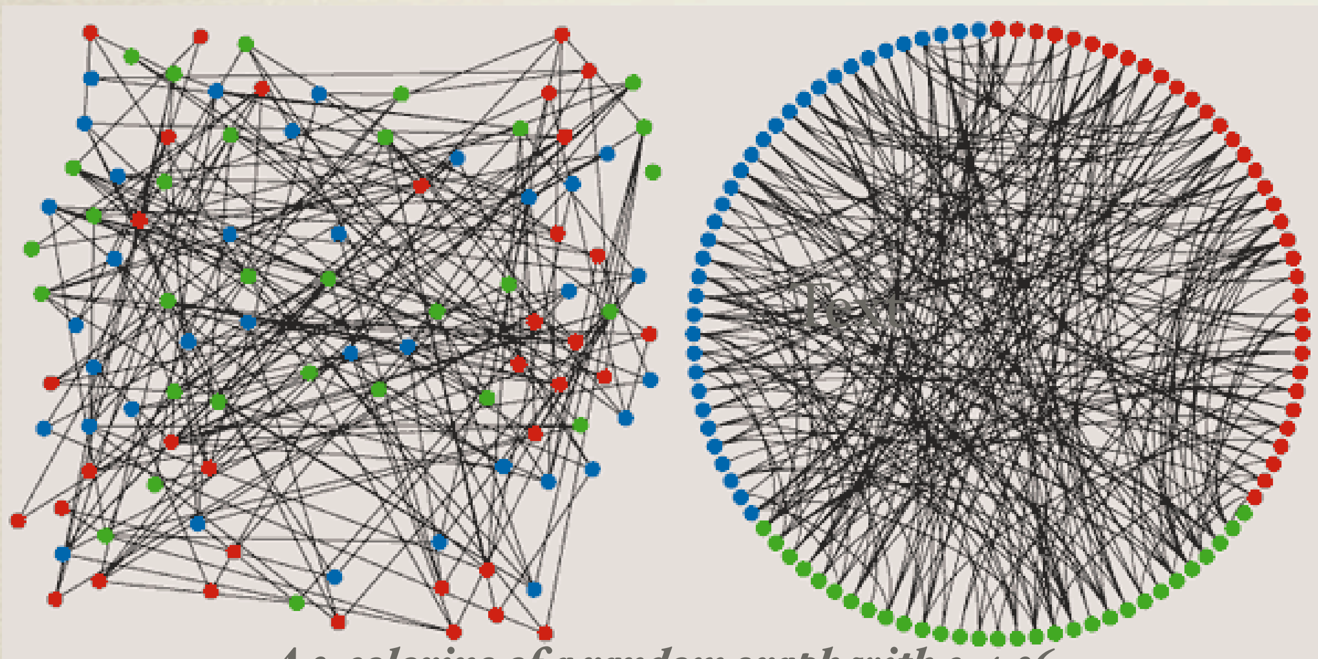

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No one has ever seen the solution of, say 5-coloring, for large enough c and $N=10^6$

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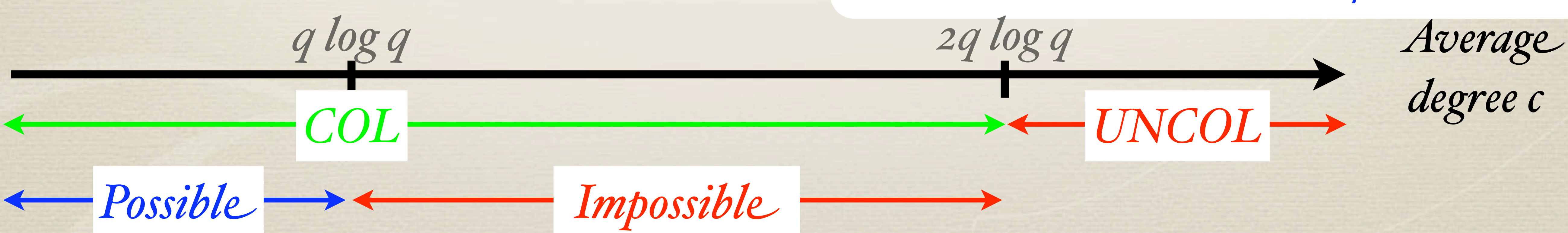
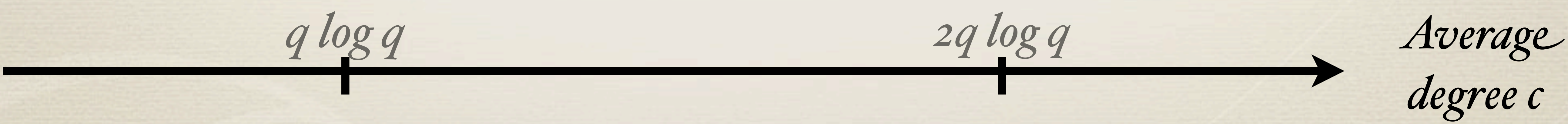


Impossible-to-simulate problems

Some optimization problems such as COL and SAT are also hard to sample



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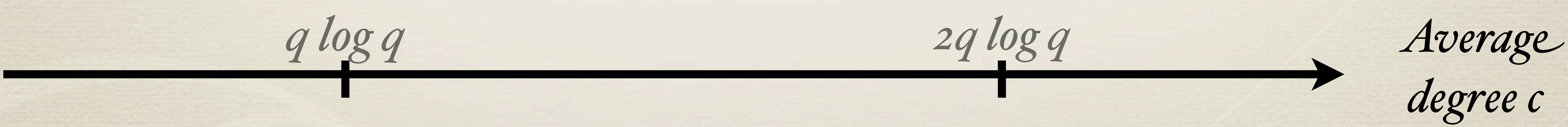
Consider the following "coloring" or "Potts-Antiferromagnet" Hamiltonian

$$\mathcal{H} = \sum_{<ij>} \delta(s_i, s_j)$$

$$s_i = 1, 2, \ldots, q$$



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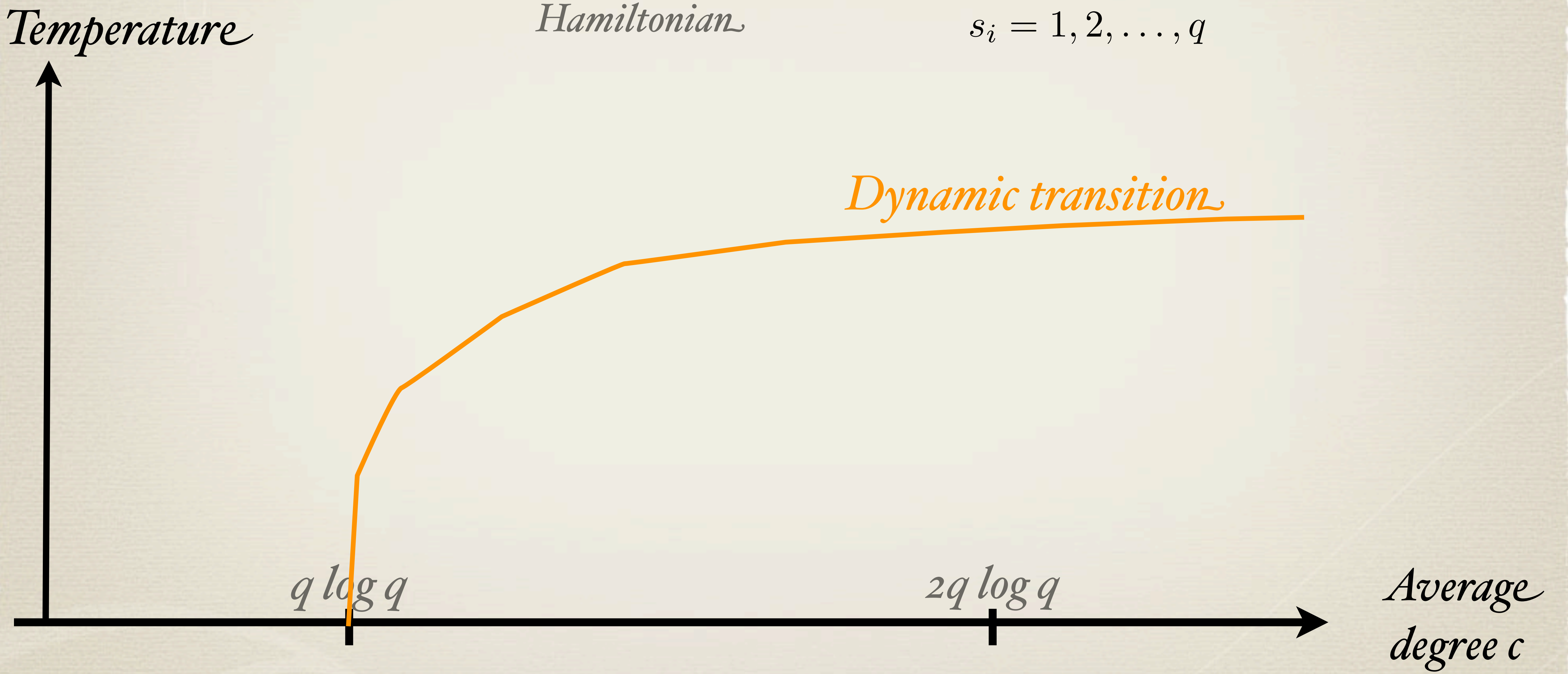

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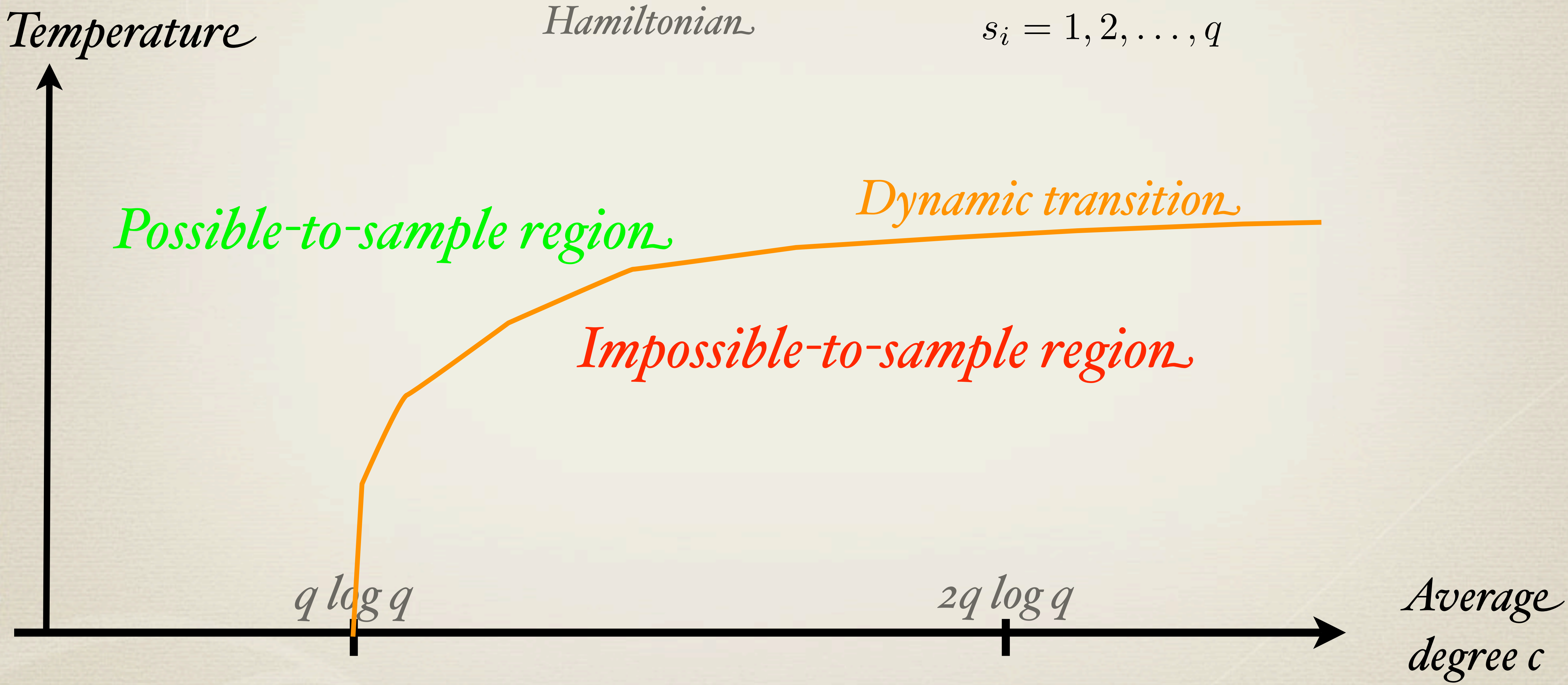

Frustrating Intractable Problems

* We <u>know</u> that some random problems DO have solutions, but we cannot find them!
* Sampling and performing Monte-Carlo is even Harder!
* Many predictions from statistical physics in random problems.... but impossible to test most of them !

A Different “Reverse” strategy

Problem → Solution

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Instead of choosing a problem, and looking for a solution….

A Different "Reverse" strategy

~~Problem → Solution~~

Solution → Problem

Instead of choosing a problem, and looking for a solution....

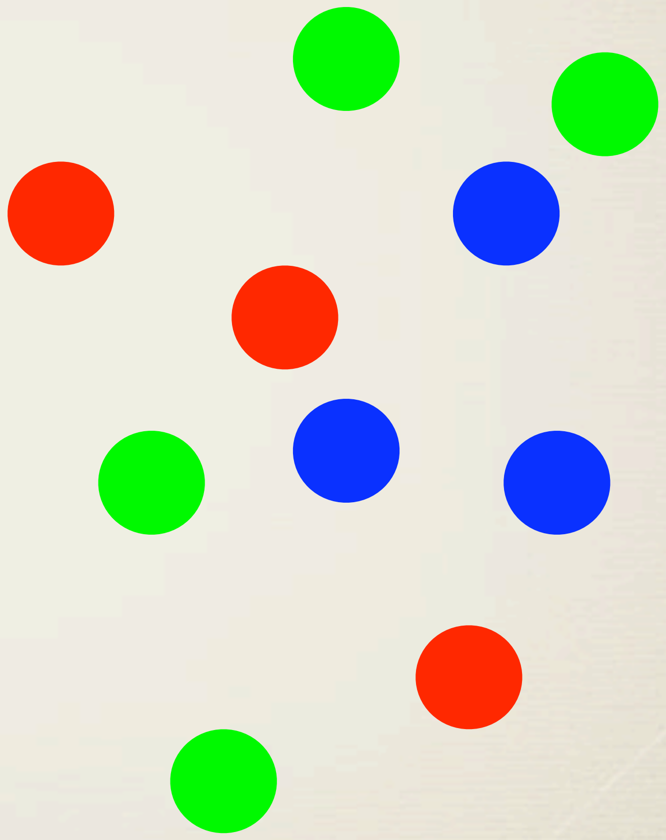
↓

We choose a configuration/assignment and
and look for a problem for which this is a solution !

The Planted Ensemble in the coloring problem

Consider the 3-coloring problem with N nodes and M links.

1) Color randomly the N nodes



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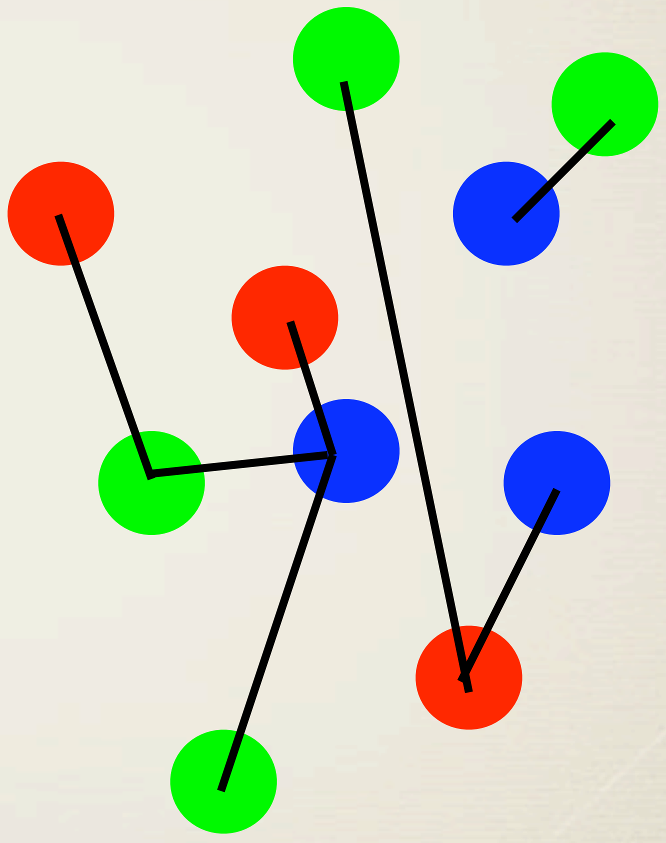
II) Put the M links randomly such that the planted configuration is a proper coloring



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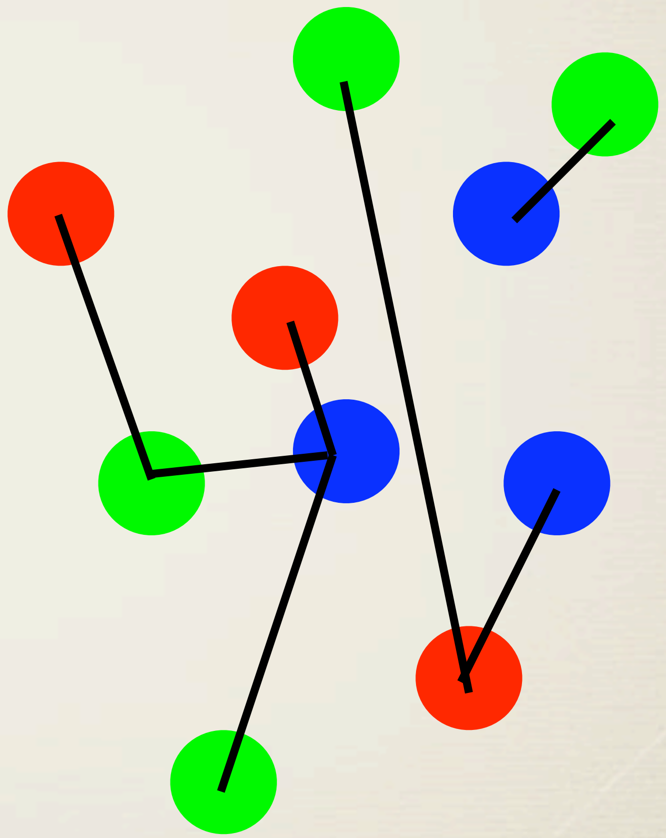
III) Now, we have created a problem for which **we know** the solution



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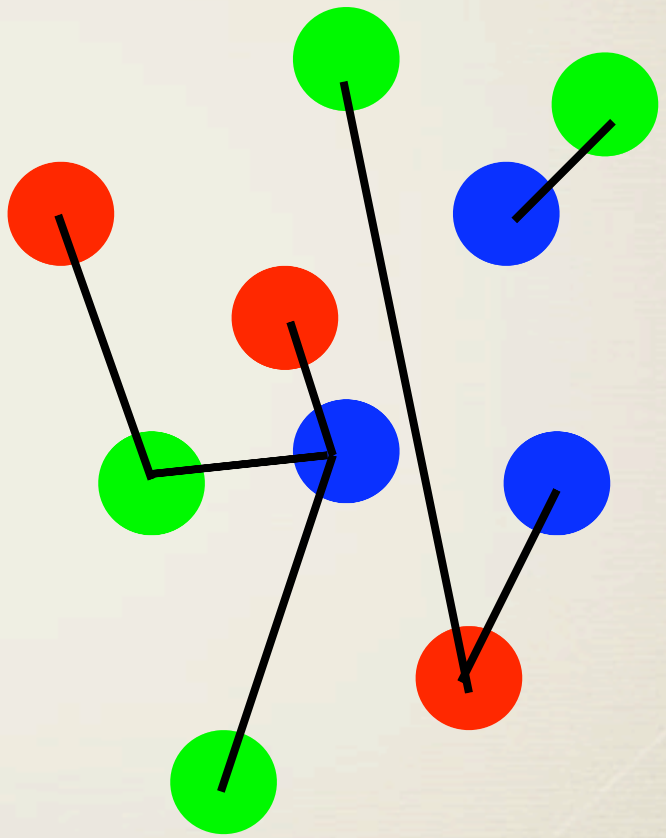

IV) We could also have prepared a configuration with a known cost/energy

The Random ensemble versus the Planted ensemble

Random ensemble

Choose a random graph with N nodes and M links

Planted ensemble

Choose a random coloring of N nodes

Choose a random graph such that this is a correct coloring...

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Montanari and Semerjian, Jstat. '06 & Achlioptas and Coja-Oghlan, arXiv:0803.2122:

The two ensembles are asymptotically ($N\to\infty$) equivalent for low enough degree c !

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Random ensemble

Choose a random graph with N nodes and M links

Planted ensemble

Choose a random coloring of N nodes

Choose a random graph such that this is a correct coloring...

Definition : **Two ensembles of random graphs are asymptotically equivalent if and only if in the thermodynamic limit every property which is almost surely true on a graph from one ensemble is also almost surely true on a graph from the other ensemble.**

Some open questions:

* Until which connectivity/degree c the planted and random ensembles are equivalent ?
* Is the planted ensemble interesting beyond this connectivity ?
* Can we generalize this approach to finite energy (coloring with a <u>finite fraction</u> of mistakes ?)
* How can we use a planted configuration ?
* What are the models where a "quiet" planting is possible ?

In this talk:

1) A (brief) summary of a theory of "quiet" planting in random models

2) Using planted configurations for fast simulations.

The Planted Ensemble

The Planted Ensemble

* We use the formalism described in Zdeborová's talk

Main result

Consider a model where the annealed computation is correct in some region (high temperature or low degree)

$$f = -\frac{1}{N}\beta[\log Z]_{dis} \quad \Leftrightarrow \quad f_{annealed} = -\frac{1}{N}\beta \log [Z]_{dis}$$

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Consider a model where a <u>factorized</u> (i.e. identical for all nodes) Belief Propagation solution is correct in some region (high temperature or low degree)

A list of models with “Quiet” planting !

This condition is fulfilled (at least in some region) for many models:

- Random q-coloring problem
- Random XOR-SAT
- Mean field spin glasses (e.g. Vianna-Bray, Sherrington-Kirkpatrick)
- Random 2-in-4 Sat
- Random Vertex-Cover (independent set)
- Any non disordered model on a random regular graphs
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This condition is not fulfilled for :

- Random K-SAT

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In the region where the two free energies are different, the planted configuration induces an additional "Gibbs" state (or BP fixed point)

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For all these models, the cavity method allows to compute the value of the threshold beyond which $f \neq f_{annealed}$

⇨"Phase transition"

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In some models, the equivalence can be proven

For all these models, the cavity method allows to compute the value of the threshold beyond which $f \neq f_{annealed}$

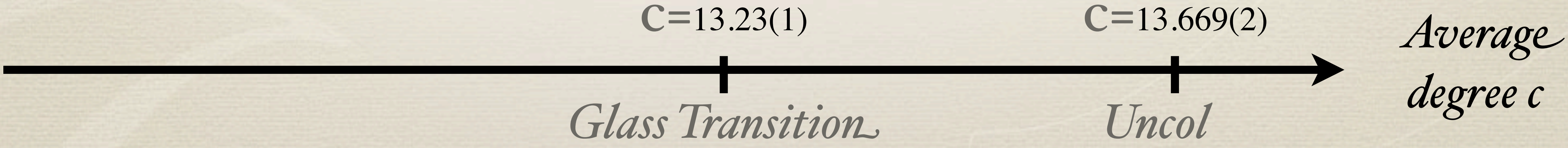
⇨"Phase transition"

Take-home message:

* <u>Conjecture 1</u>: the planted model is equivalent to the original one up to the point where the annealed solution is correct
(for physicists: up to the static <u>spin glass</u> transition...)
and the planted configuration is a "<u>typical</u>" one.

A Solution To an Impossible-To-Solve Problem

ex: 5-coloring of Erdös-Renyi random graphs



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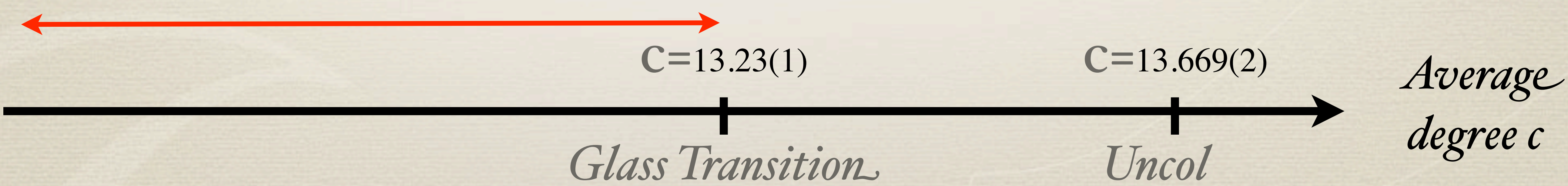

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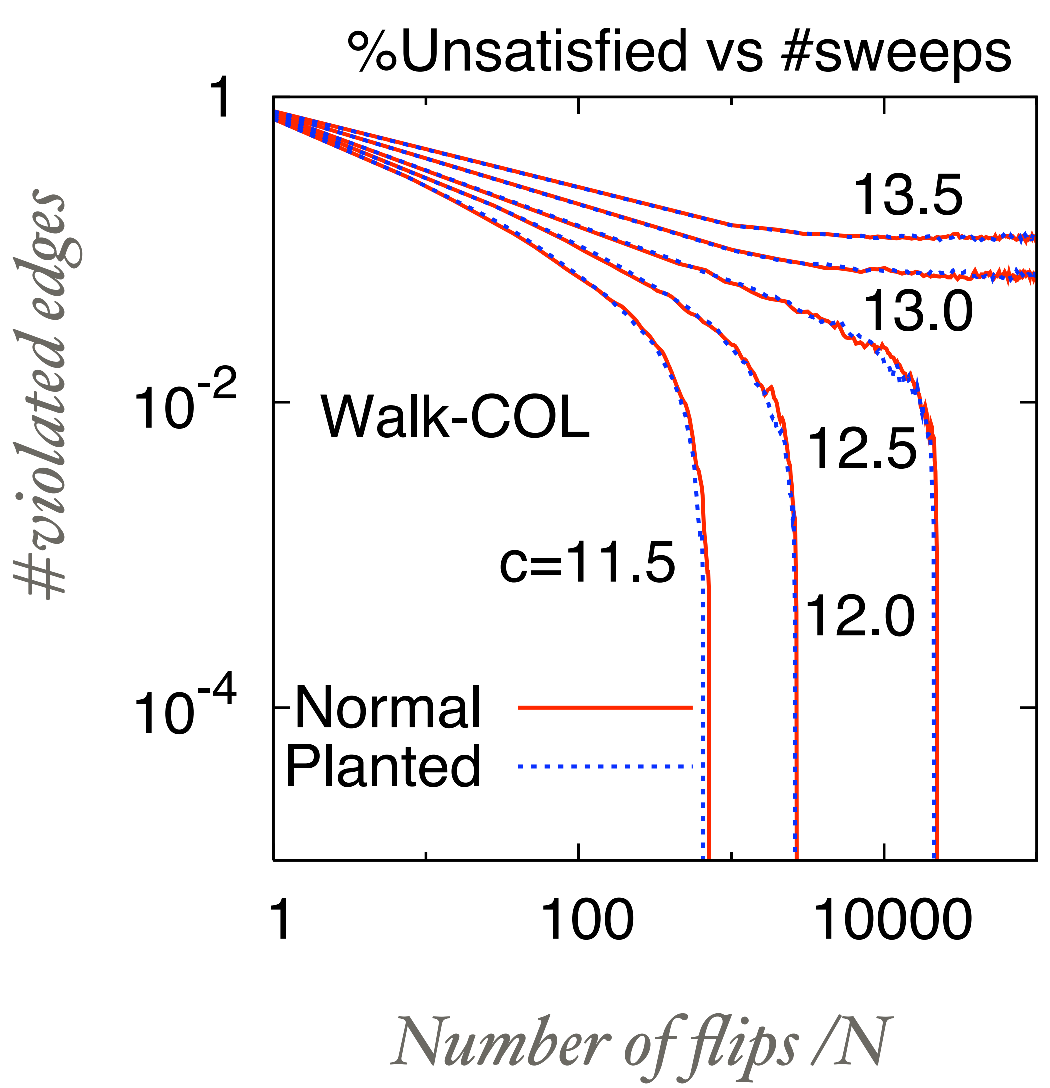
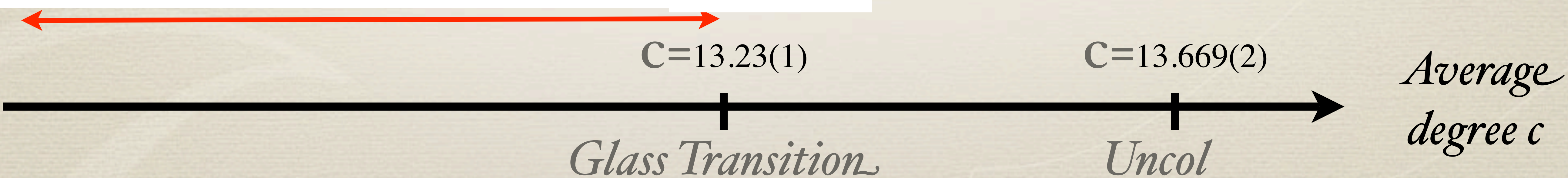
5-coloring using walkcol with $N=10^6$



One can create impossible to solve problems of any size where the solution is known only by the creator



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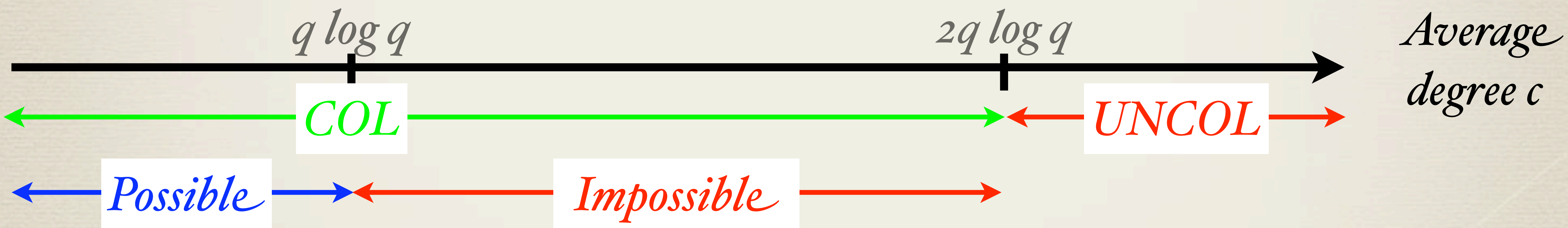

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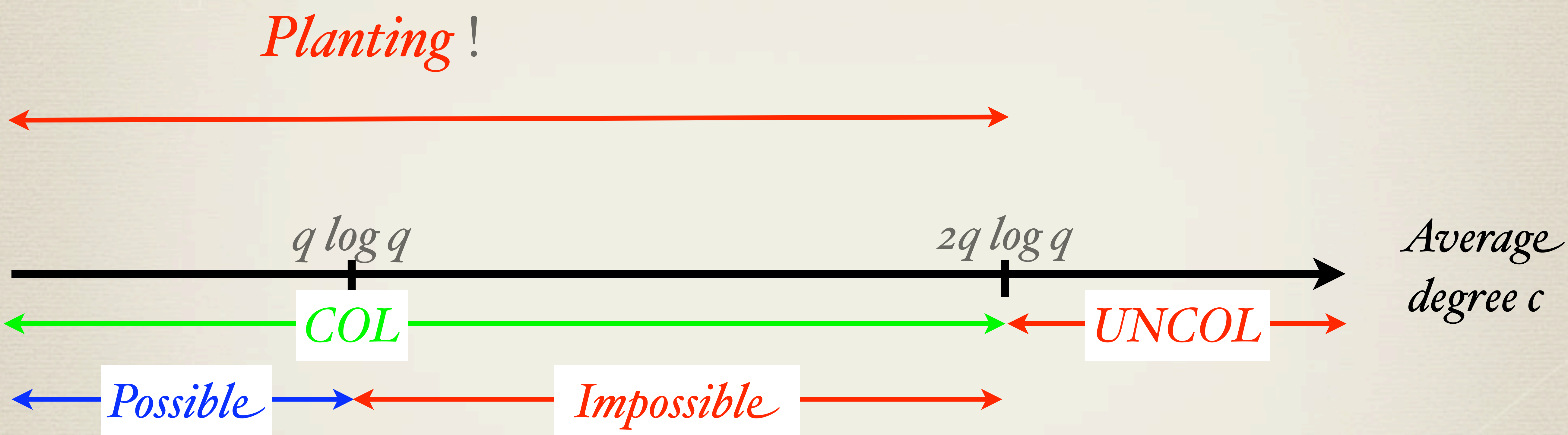

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  (for physicists: up to the spin glass transition…)
  and the planted configuration is a “typical” one

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U. Feige, E. Mossel and D. Vilenchik.
Proceedings of Random'06, LNCS 4410,

Planted configuration easy to find for large enough c

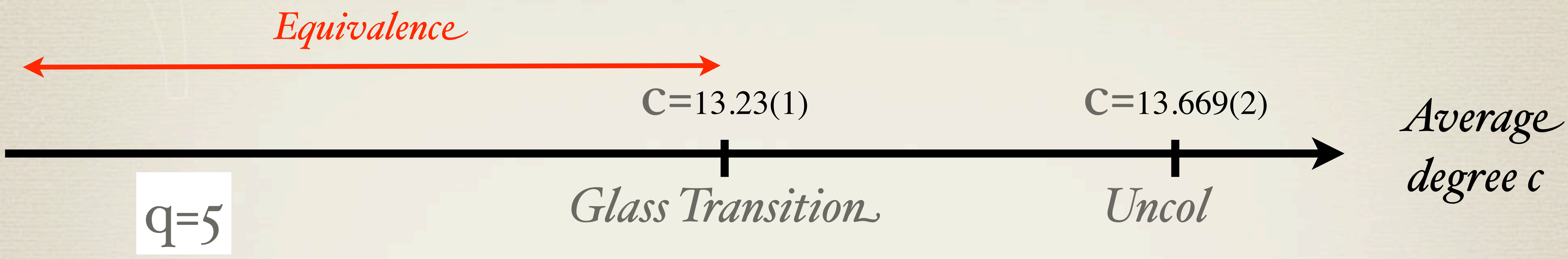
Take-home message:

* Conjecture 1: the planted model is equivalent to the original one up to the point where the annealed solution is correct
  (for physicists: up to the spin glass transition...)
  and the planted configuration is a "typical" one

* Conjecture 2: Planted configuration are hard to find until the so-called Kesten-Stigum threshold,
  (for physicists: this is the local spin glass instability)
  beyond which they can be solved easily using BP.

Example in coloring

The planted solution is well hiden until $c_{KS}=(q-1)^2$



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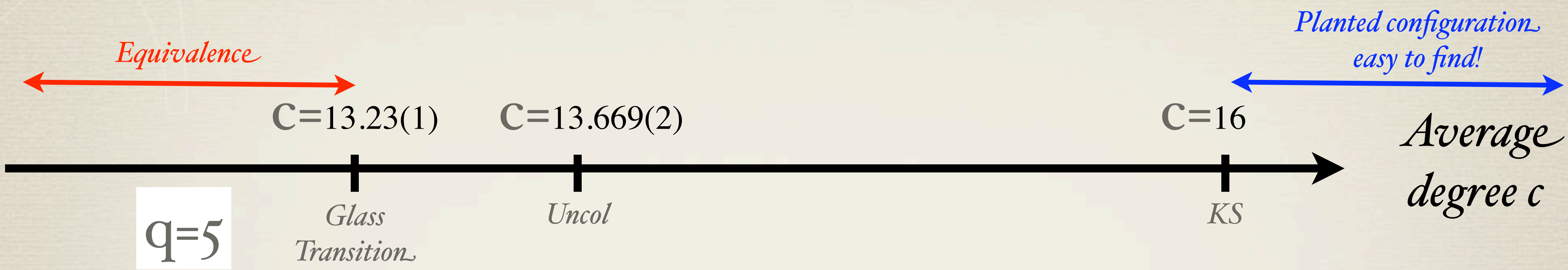

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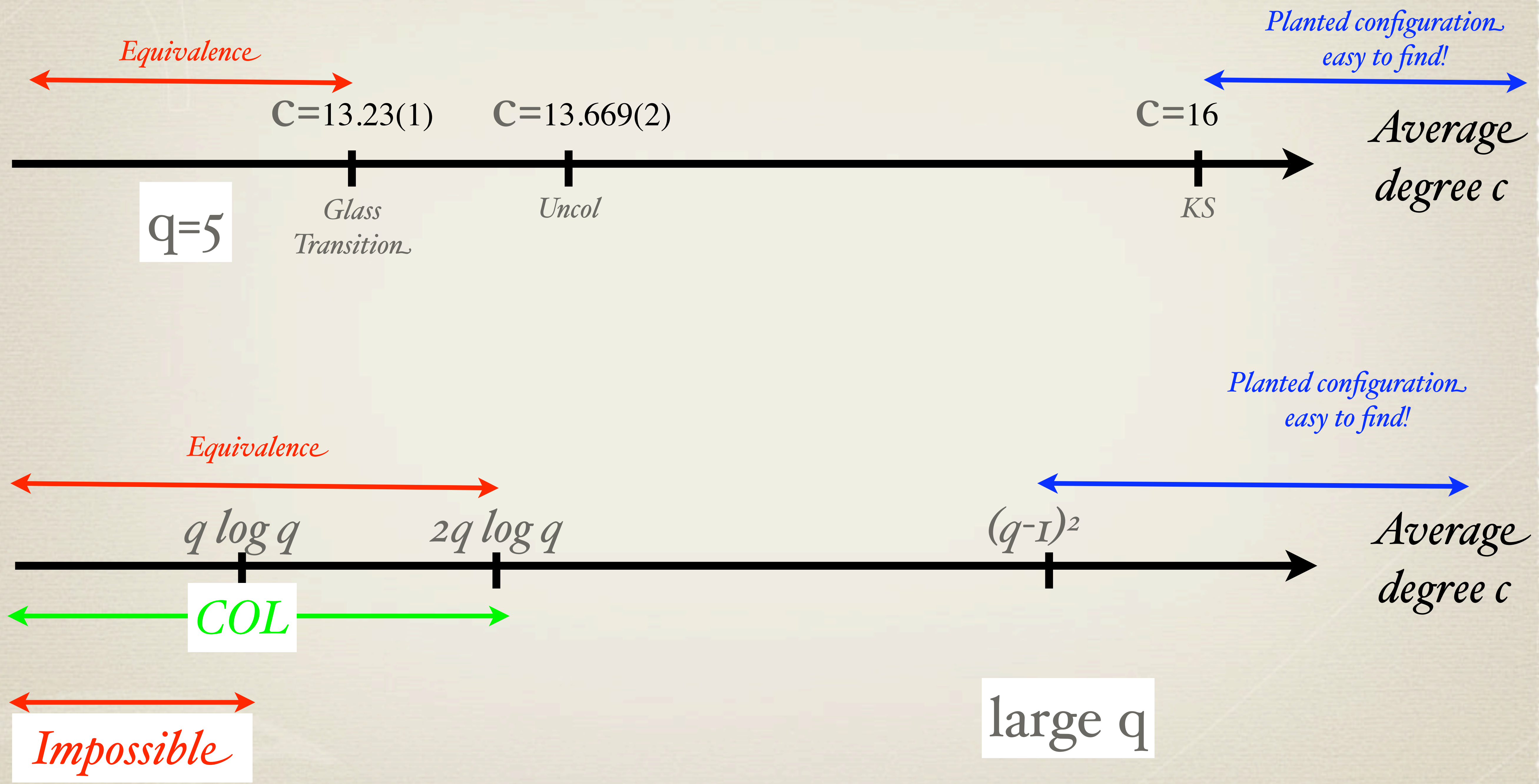
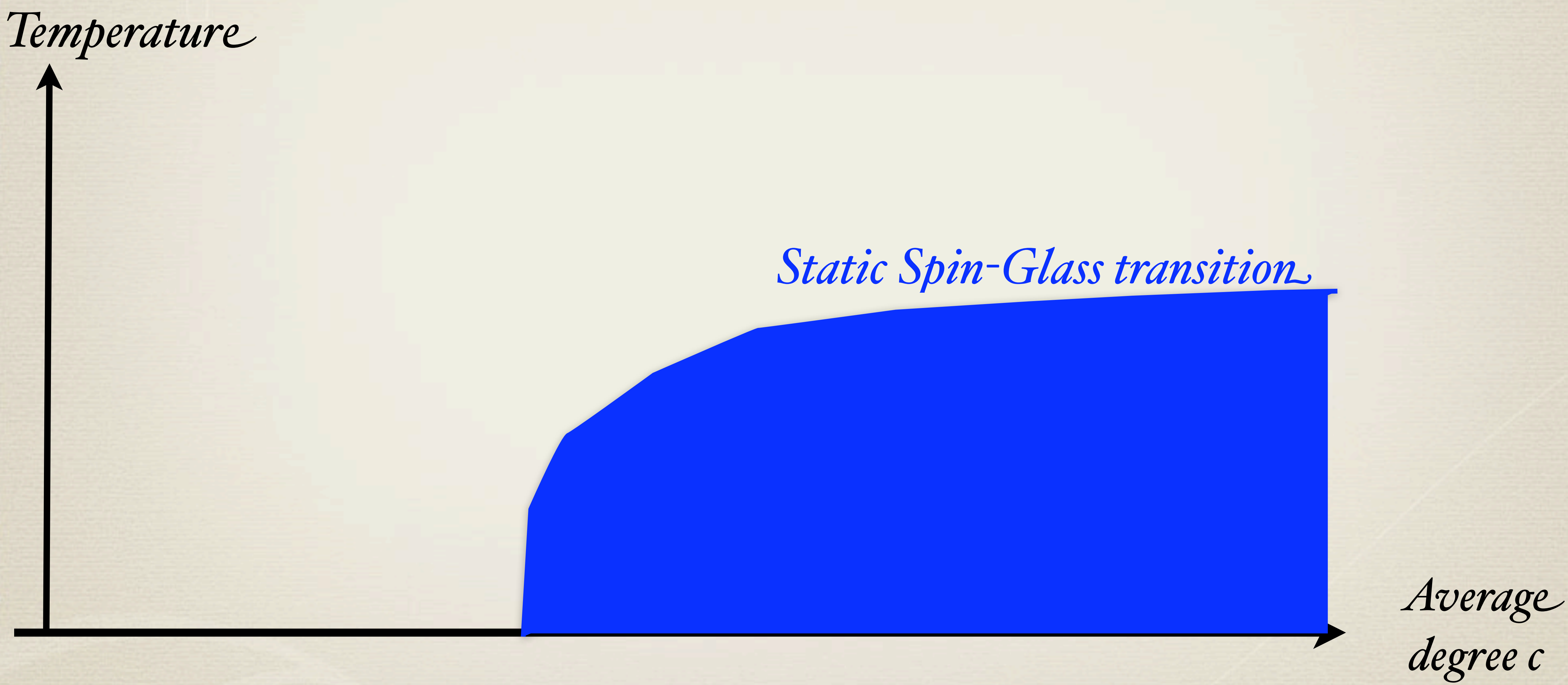


Simulating “impossible-to-simulate” models

How to perform simulations that
are usually considered to be impossible?

Impossible-to-simulate problems

Random optimization problems & mean-field spin glasses



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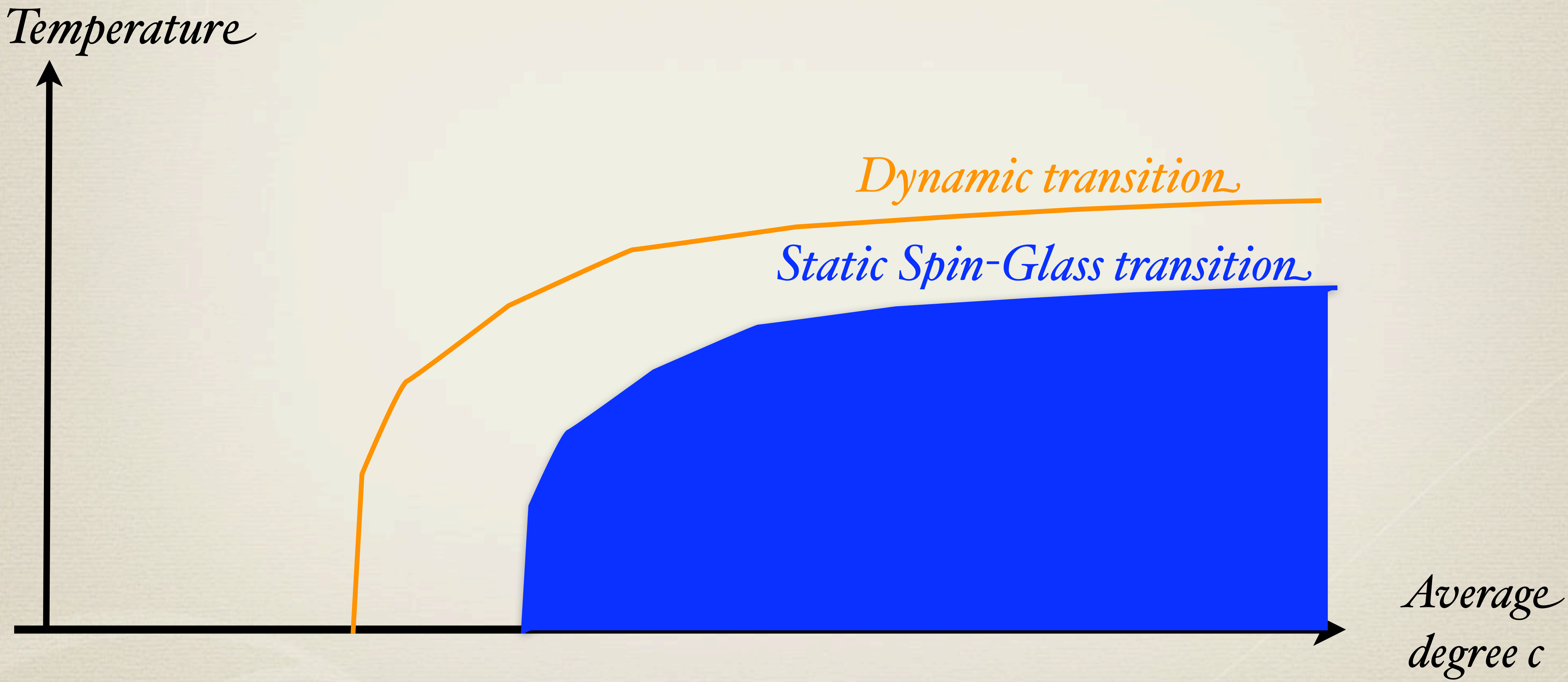

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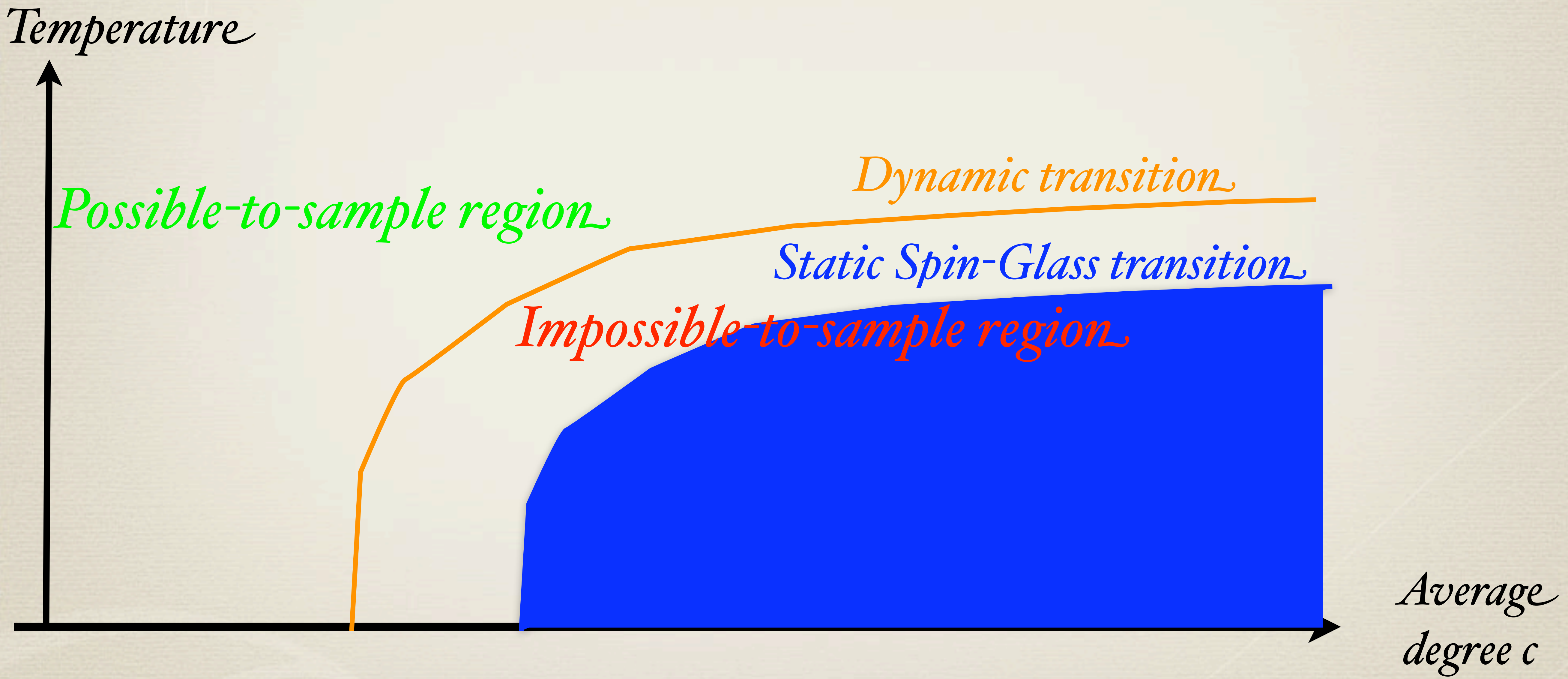
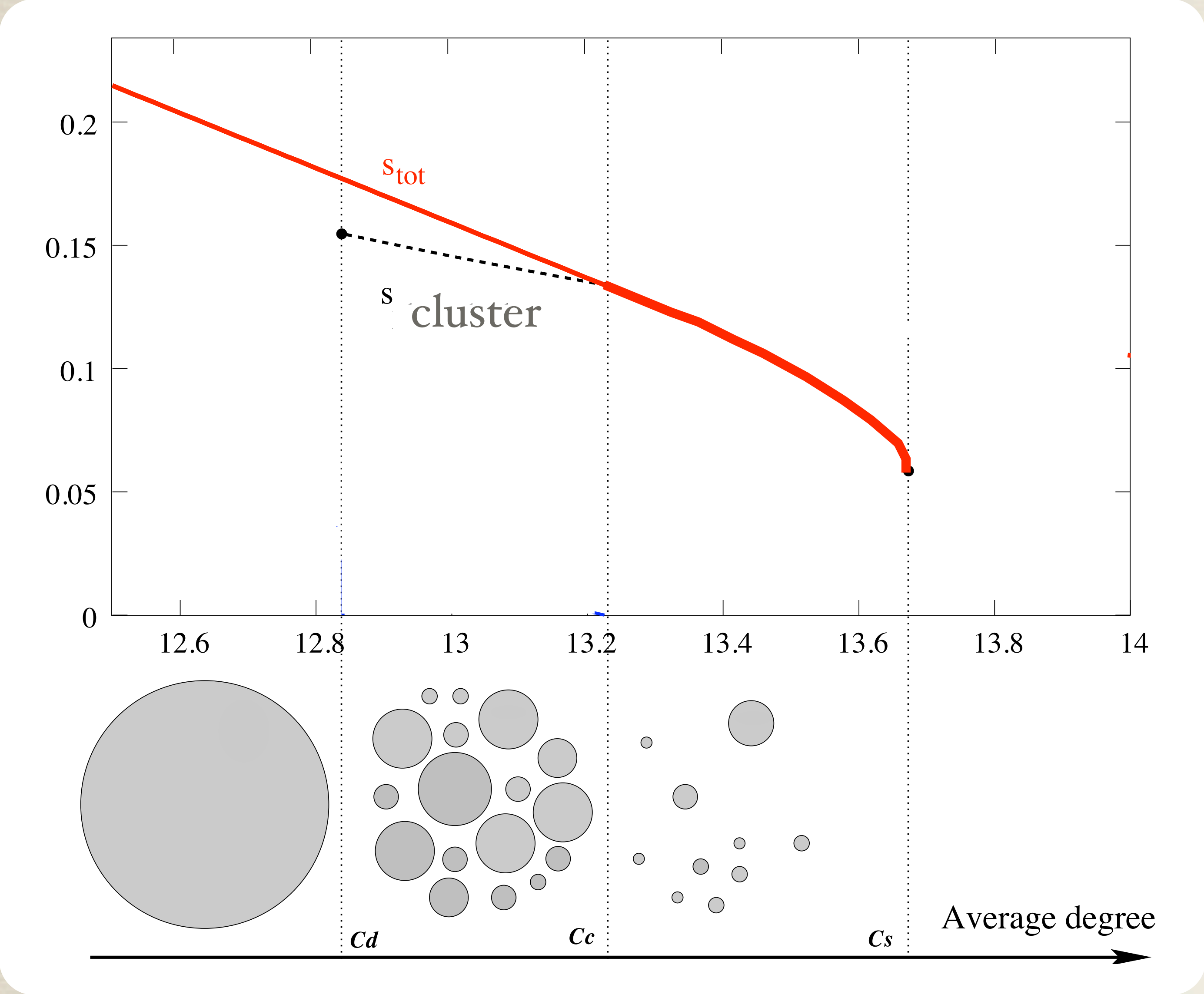


Modus operandi

1. Plant a configuration, create the graph such that the configuration satisfies all constraints
2. We now have a random instance and a “typical” equilibrium solution at zero temperature
3. We use it !

Example 1:

Testing the cavity predictions for the clustering transition



$$\psi_{factorized} = \left(\frac{1}{q}, \frac{1}{q} \dots \frac{1}{q}\right)$$

$$M = \frac{1}{qN} \sum_{graph} \sum_{c=1}^{q} \frac{\psi_{BP}^{c,i} - \frac{1}{q}}{1 - \frac{1}{q}}$$

FK, Montanari, Semerjian, Ricci-Tersenghi, Zdeborova, PNAS 07 & FK and Zdeborova, PRE 07

Prediction: beyond the so-called "dynamic" threshold, a non-trivial non-factorized fixed point of BP is obtained if one starts from an equilibrium configuration

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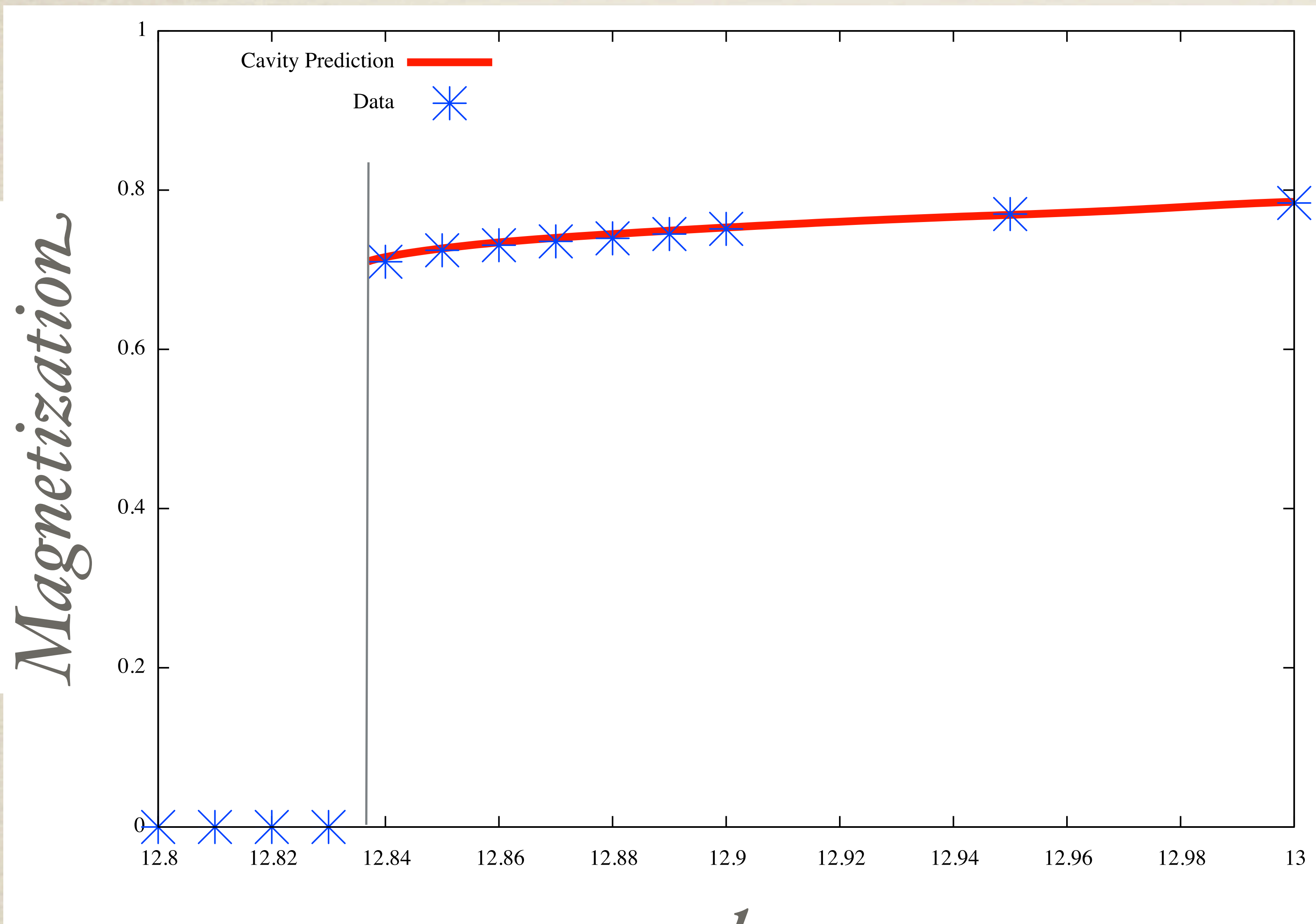
Magnetization



average degree

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Simulation with N=10⁶

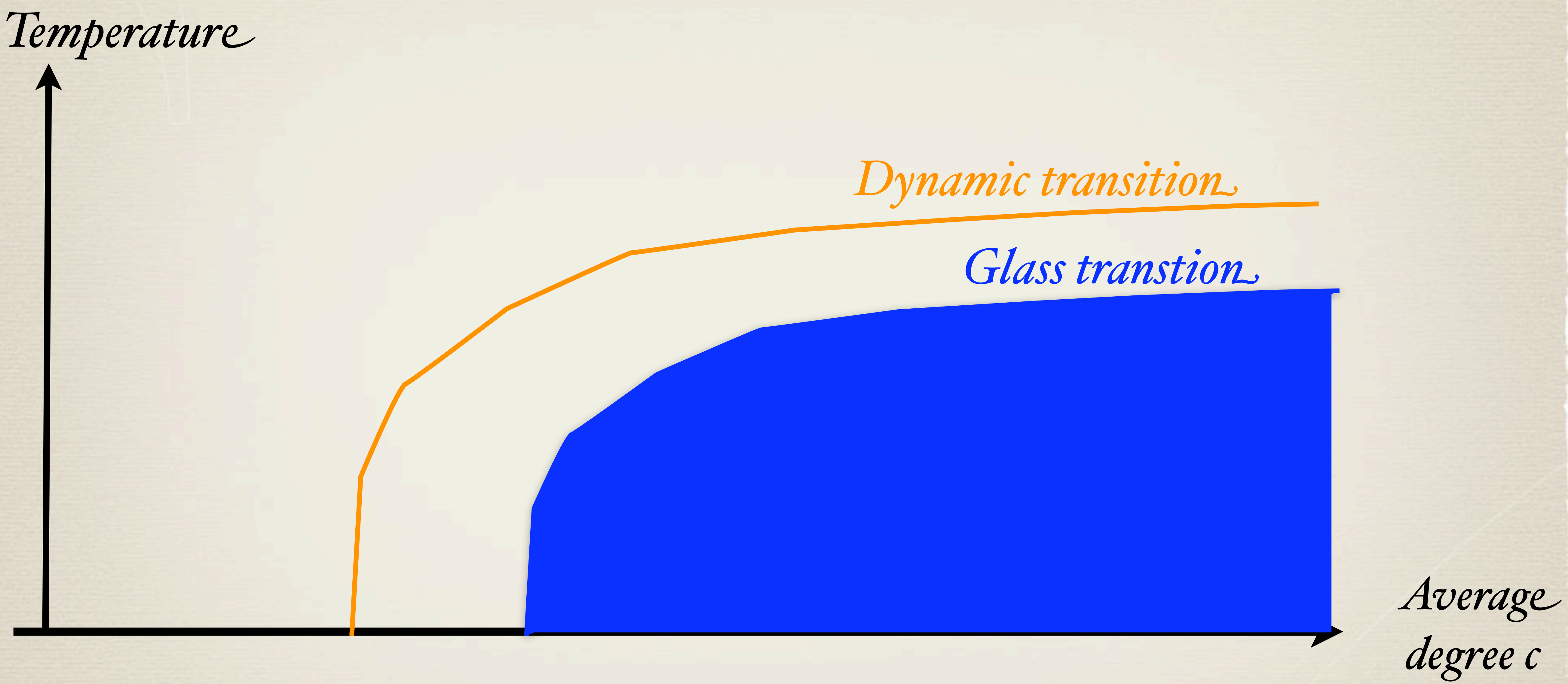
Prediction: beyond the so-called “dynamic” threshold, a non-trivial non-factorized fixed point of BP is obtained if one starts from an equilibrium configuration

Modus operandi for finite temperature simulations

1. Plant a configuration, create the graph such that the configuration has exactly the equilibrium energy
2. We now have a random instance and a "typical" equilibrium solution at temperature T
3. We use it !

Example 2:

Testing the cavity predictions for the clustering transition



<u>Prediction</u>: beyond the so-called "dynamic" threshold, the Monte-Carlo Dynamic is trapped!

Ex: 3-XORSAT, T=0.255,c=3

Example 2:

Testing the cavity predictions for the clustering transition

Usual Approach:

1) Start with a random initial condition

2) compute the correlation function:

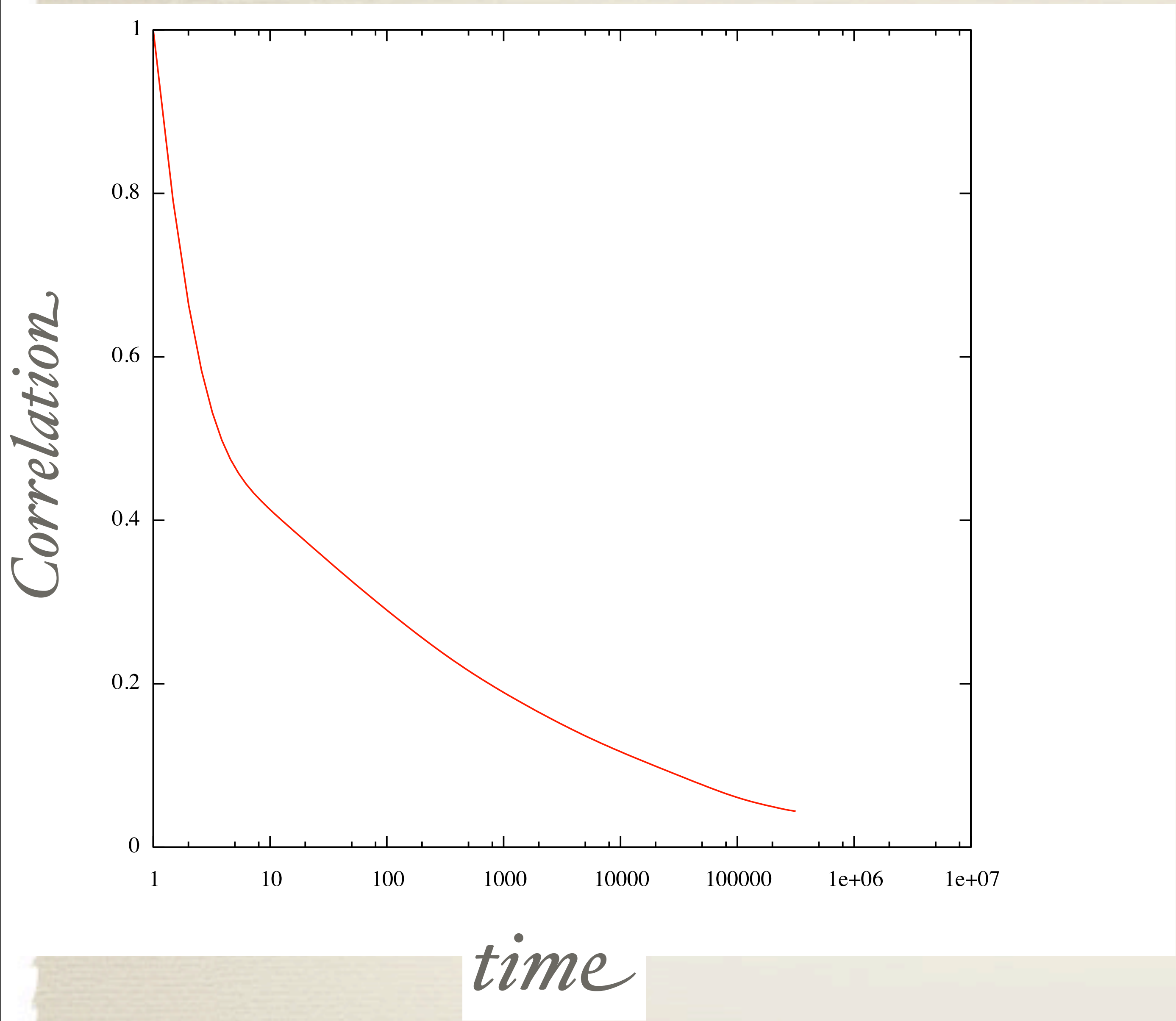
$$C(t) = \frac{1}{N} \sum_{i=1}^{N} S_i(t_{init} = 0) S_i(t)$$

<u>Prediction</u>: beyond the so-called "dynamic" threshold, the Monte-Carlo Dynamic is trapped!

Ex: 3-XORSAT, T_d=0.255

Example 2:

Testing the cavity predictions for the clustering transition



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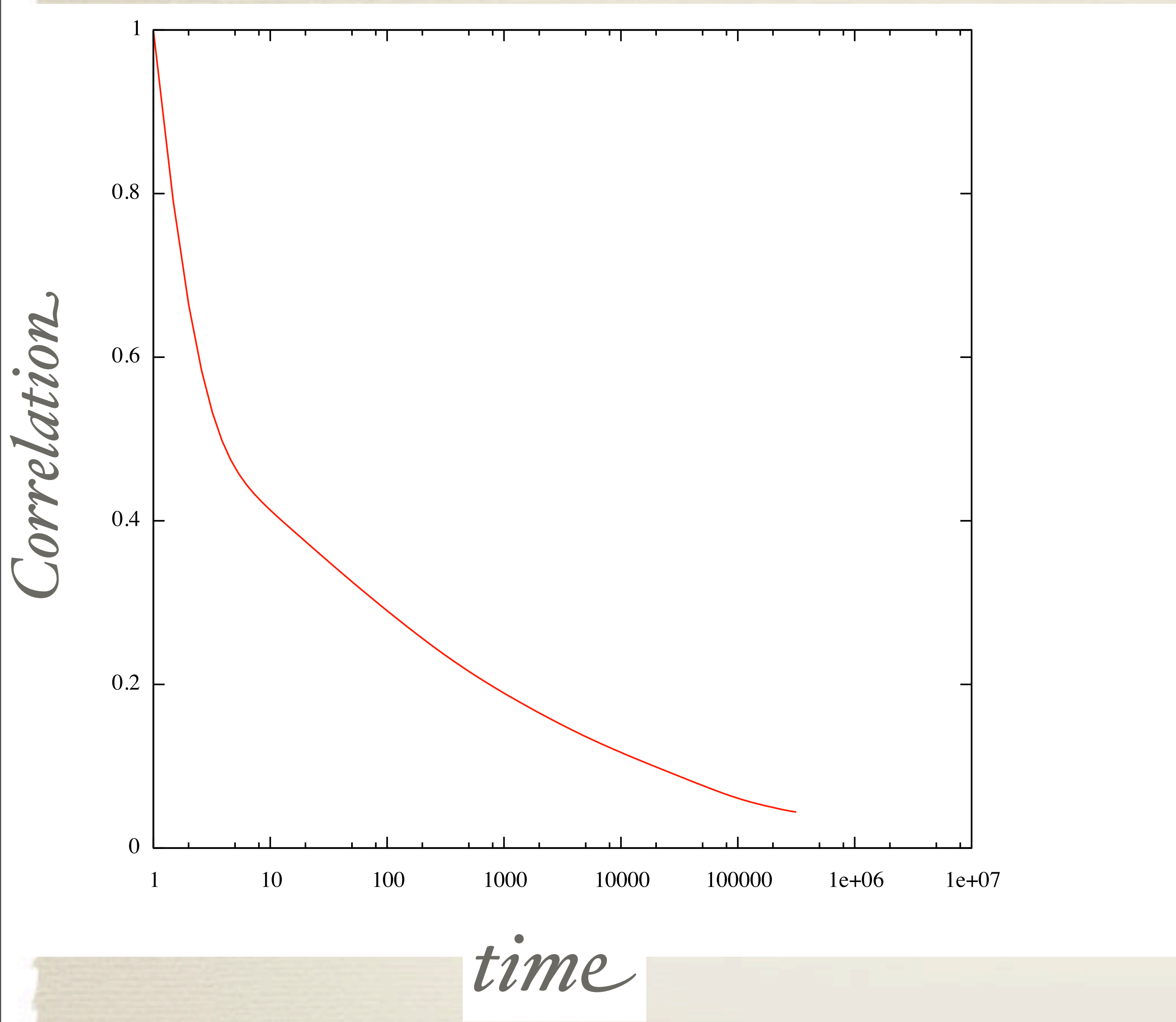

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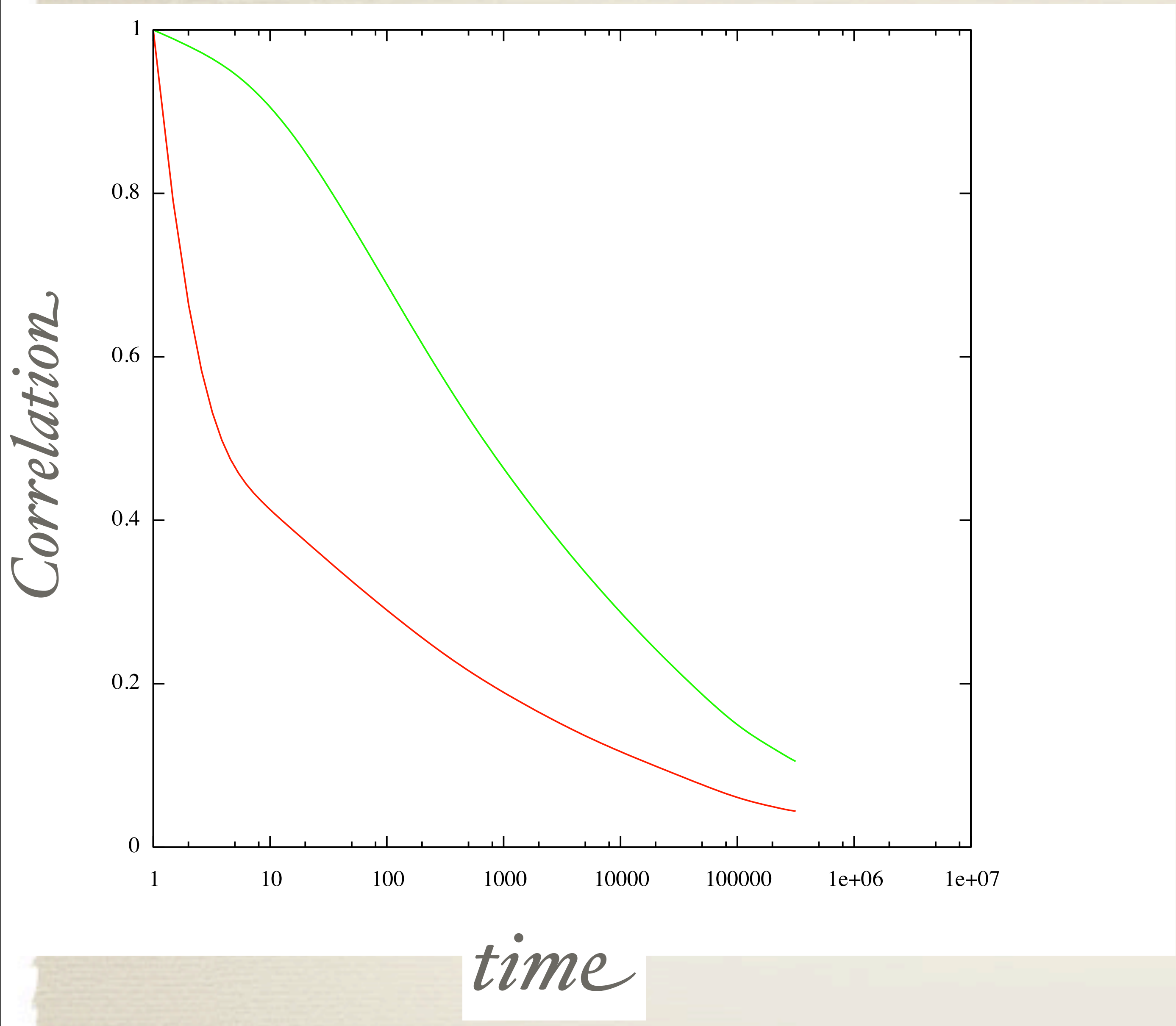

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tw=10

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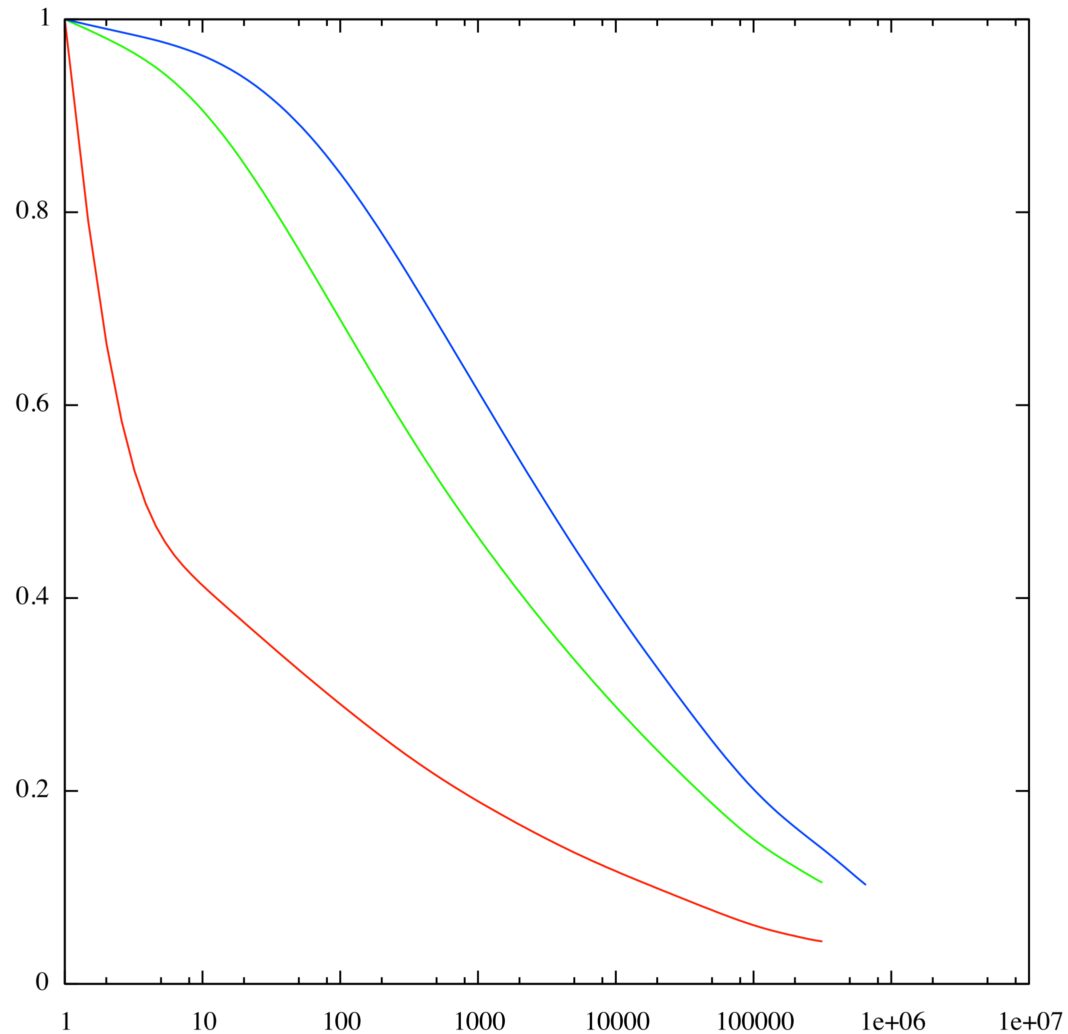

Correlation

time

Usual Approach:

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tw=100

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Ex: 3-XORSAT, T_d=0.255

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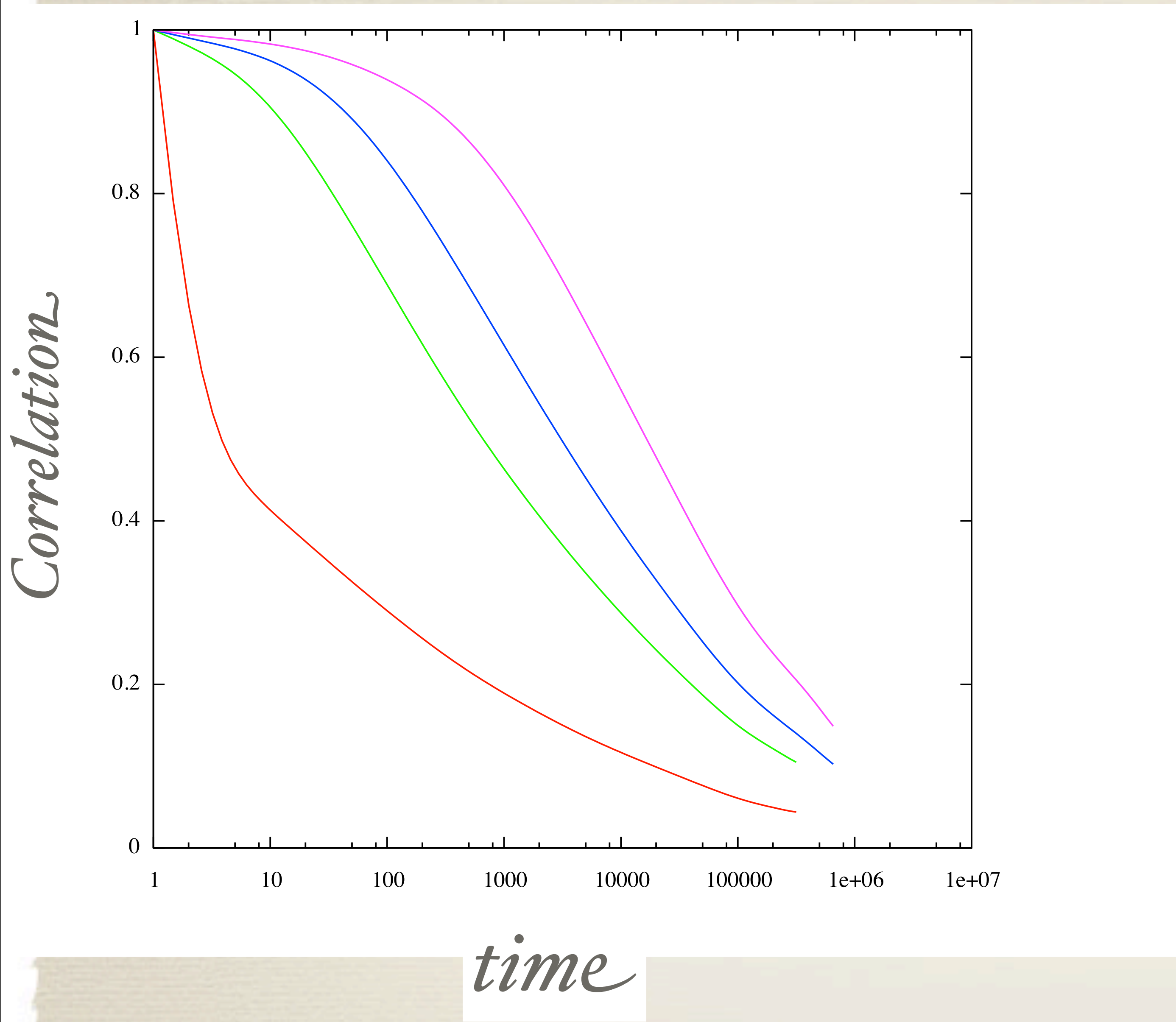
Testing the cavity predictions for the clustering transition



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tw=1000

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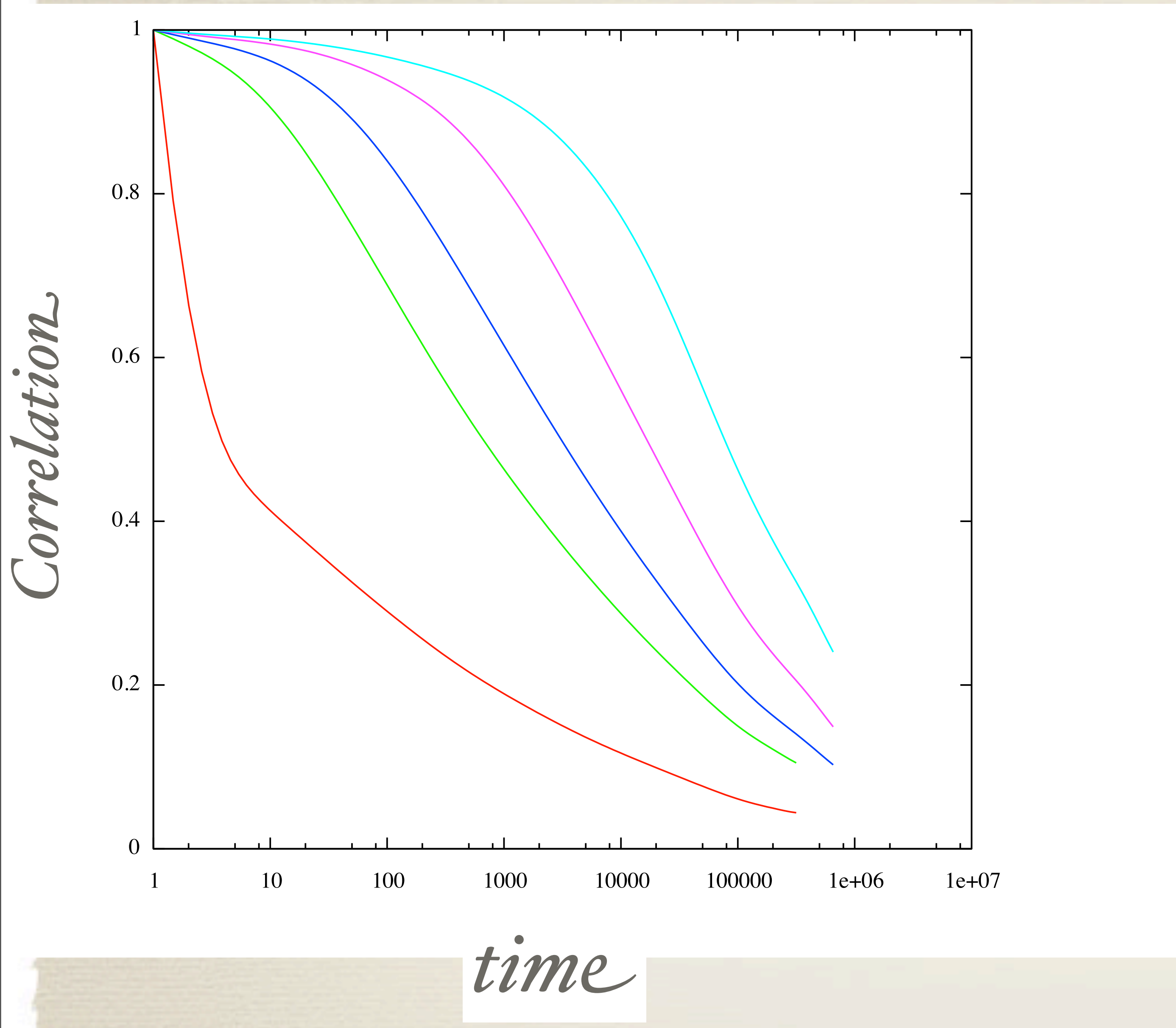

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tw=10000

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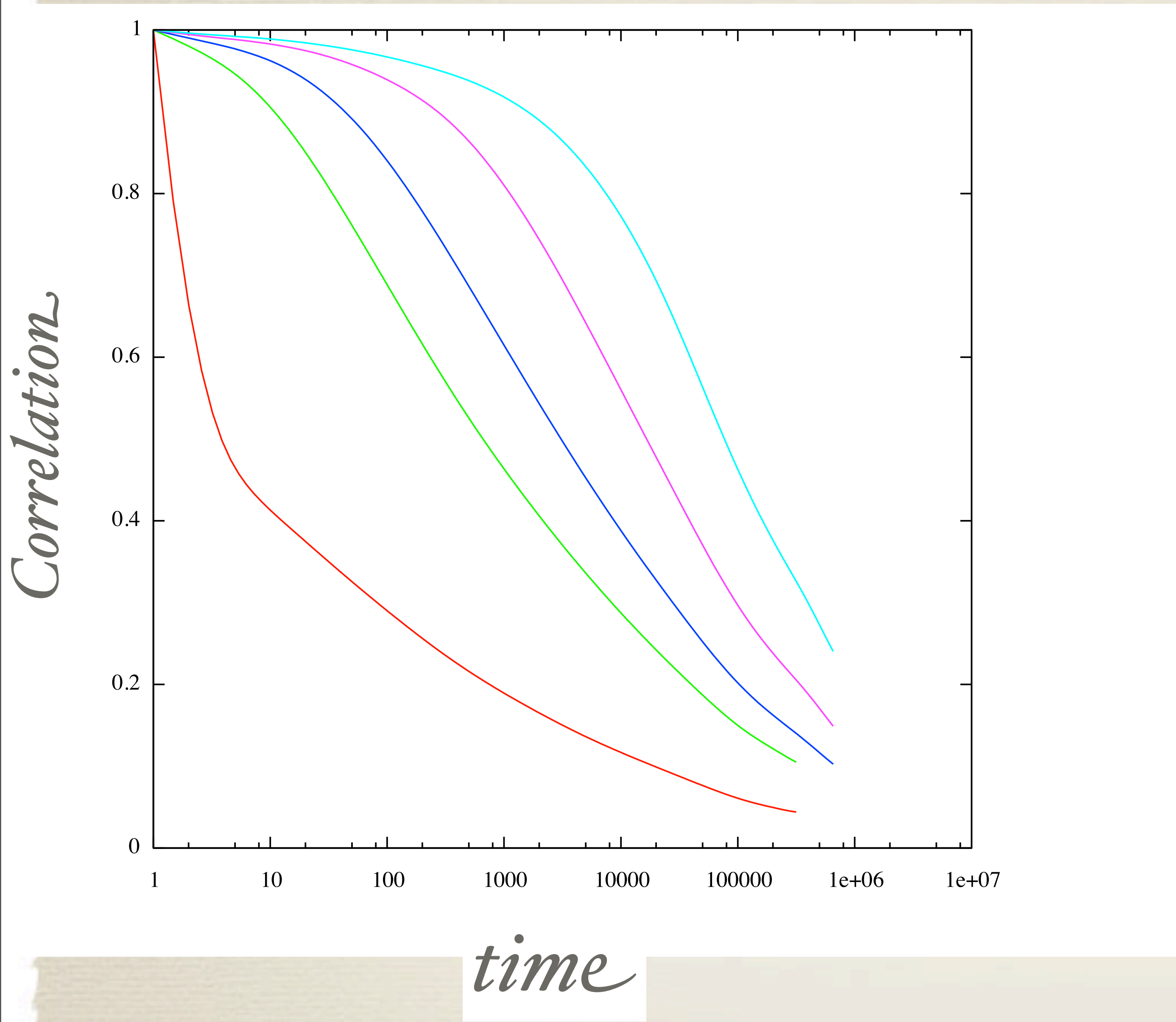

A better Approach:

1) Start with an <u>equilibrated</u> initial condition

2) compute the correlation function:

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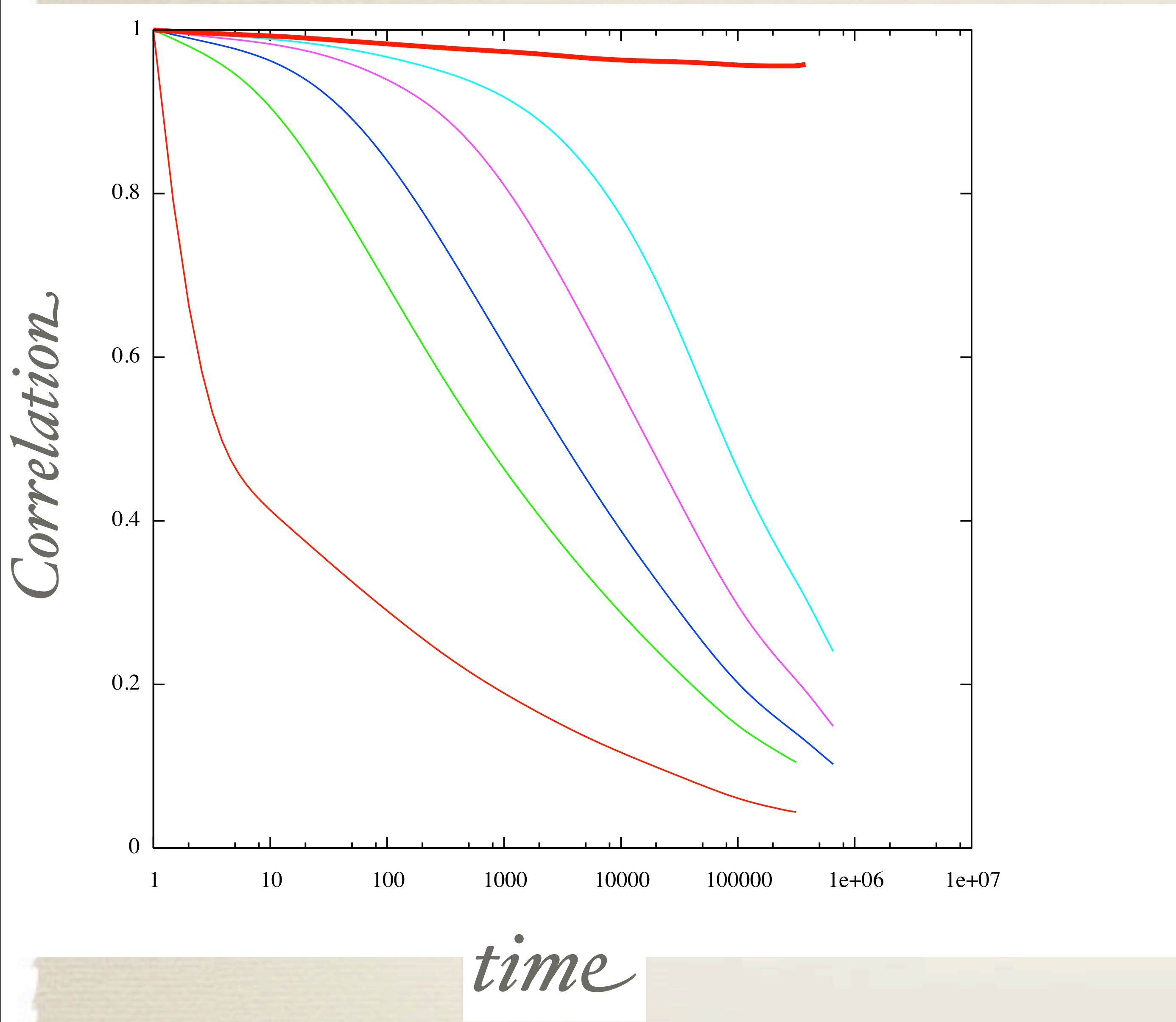

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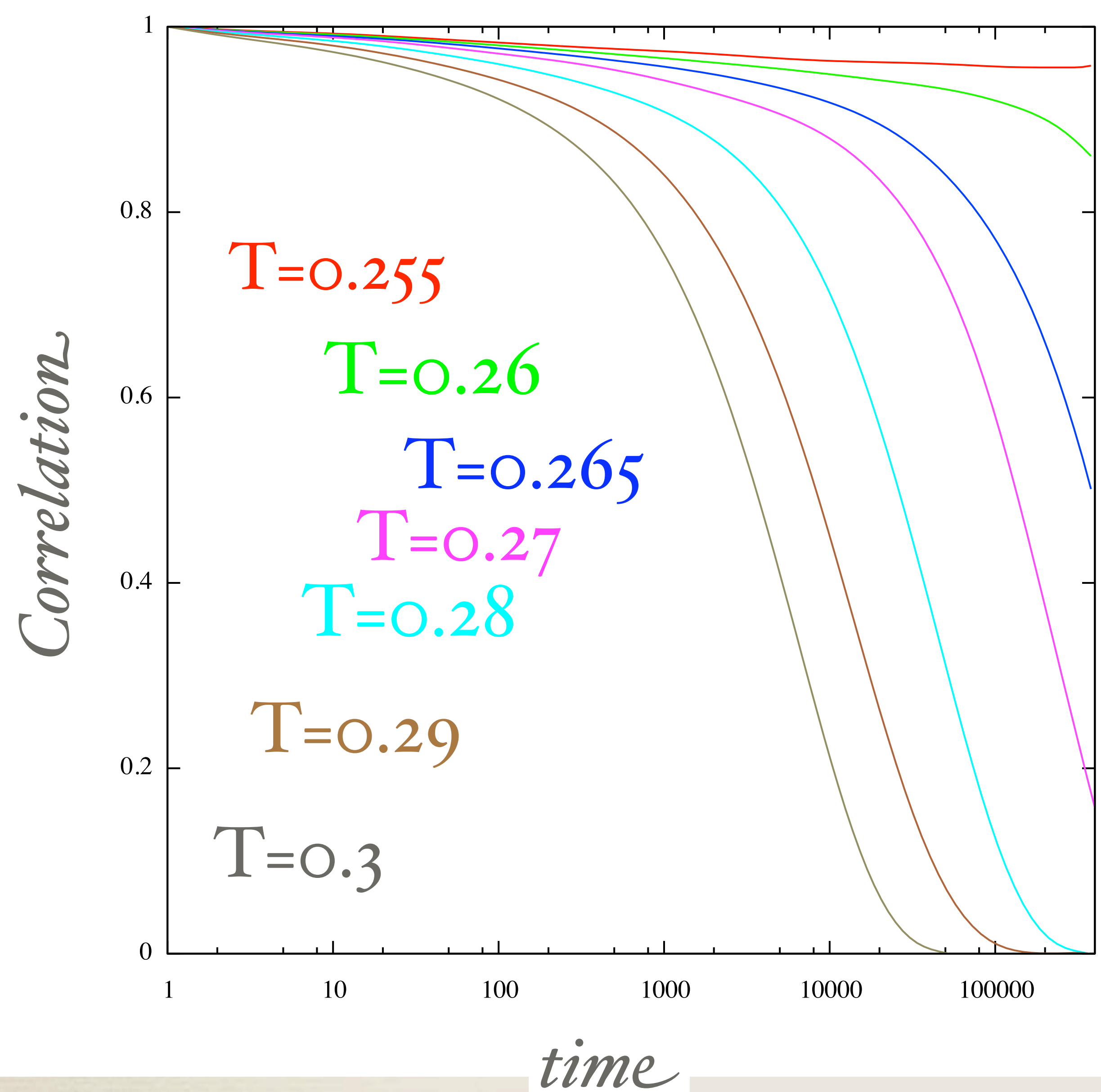

A better Approach:

Start with an equilibrated initial condition
Many temperatures:

Divergence of the relaxation time

Prediction: beyond the so-called "dynamic" threshold, the Monte-Carlo Dynamic is trapped!

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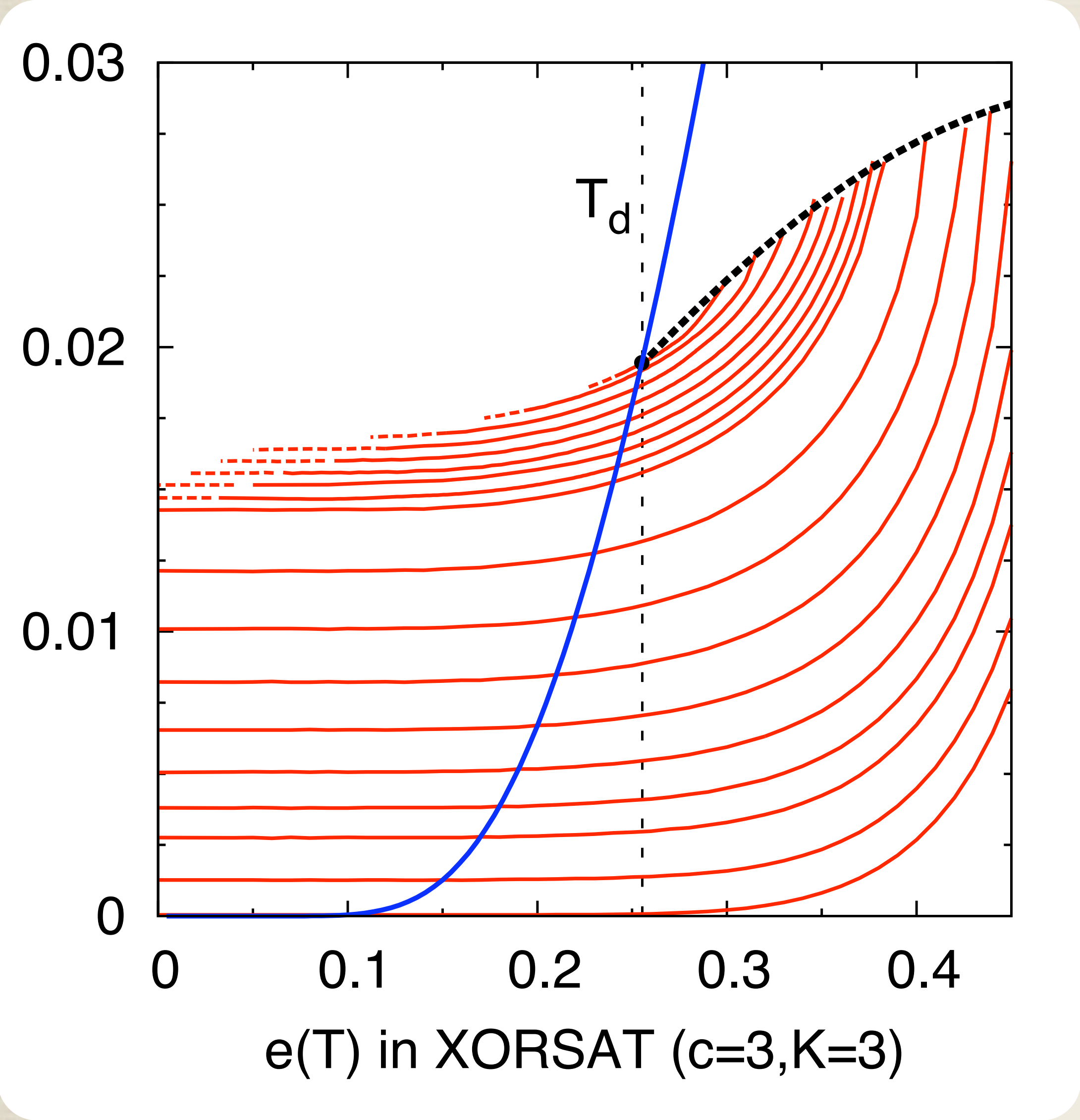
Example 3:

Studying Monte-Carlo annealings starting from equilibrium



XOR-SAT problems
(Parity-check)

$$\mathcal{H}(\{S\}) = \sum_{ijk} \frac{1 + J_{ijk} S_i S_j S_k}{2}$$

Prediction: cf. Zdeborová's talk:
Monte Carlo cooling and heating follow a well defined line

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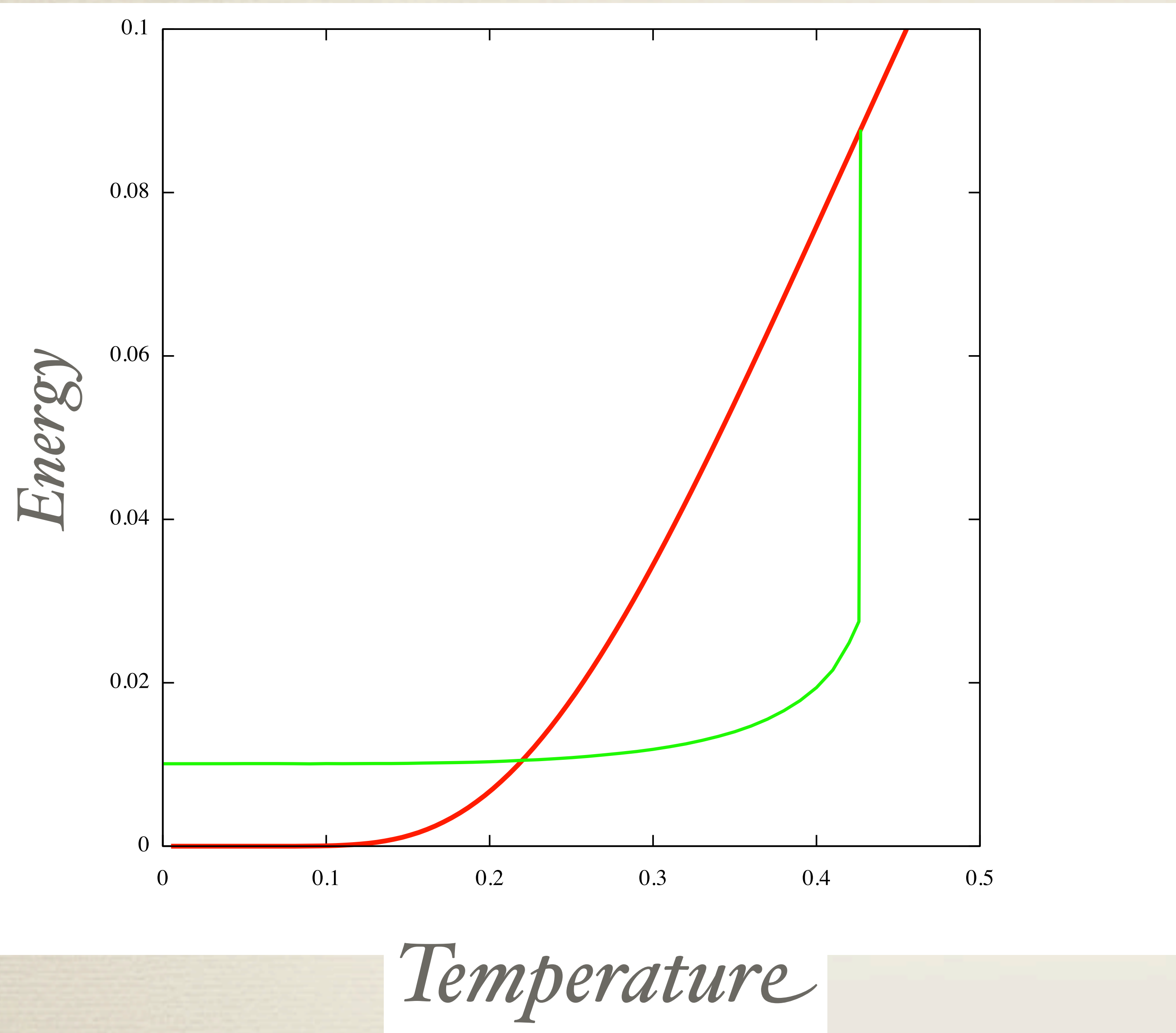

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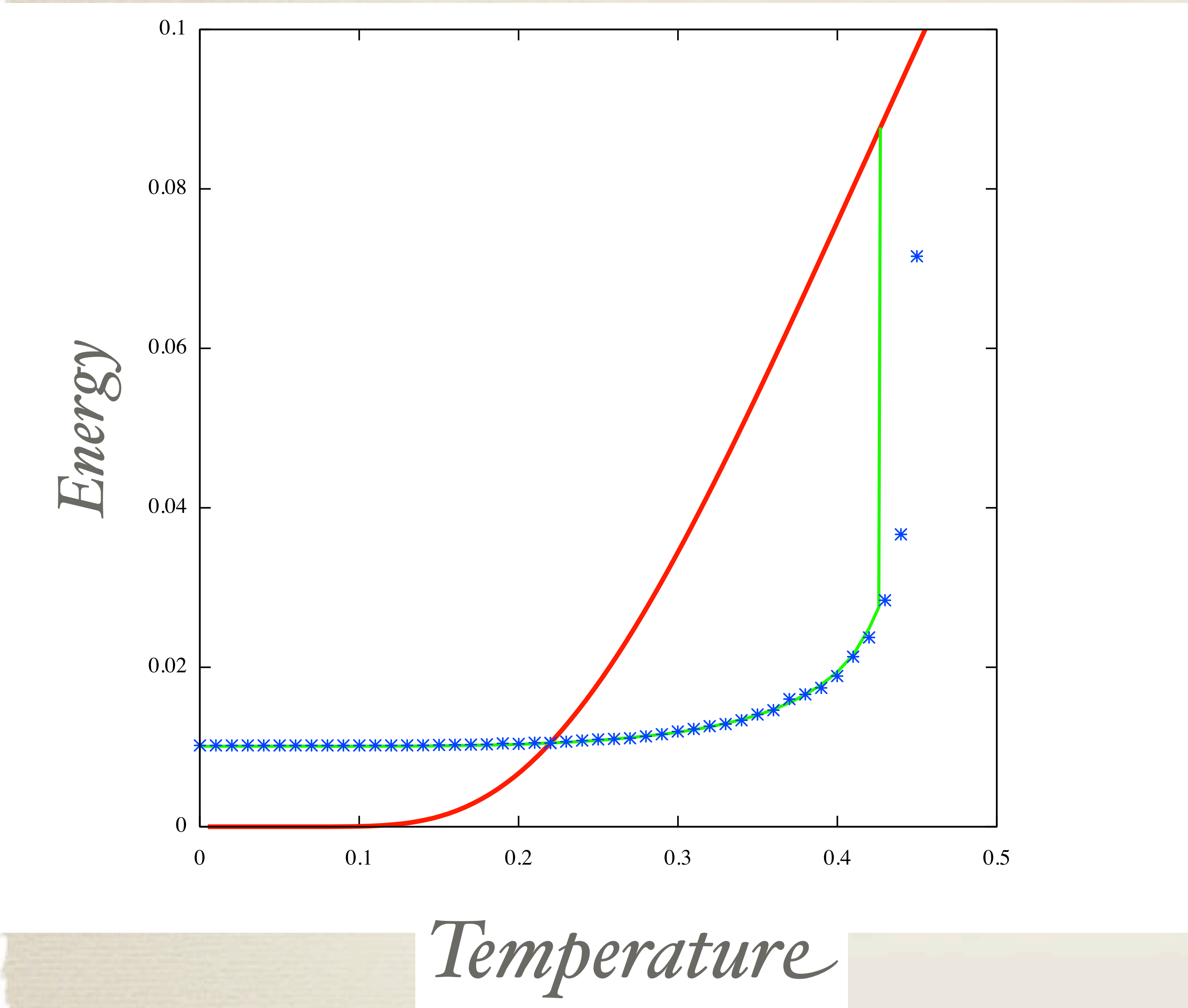

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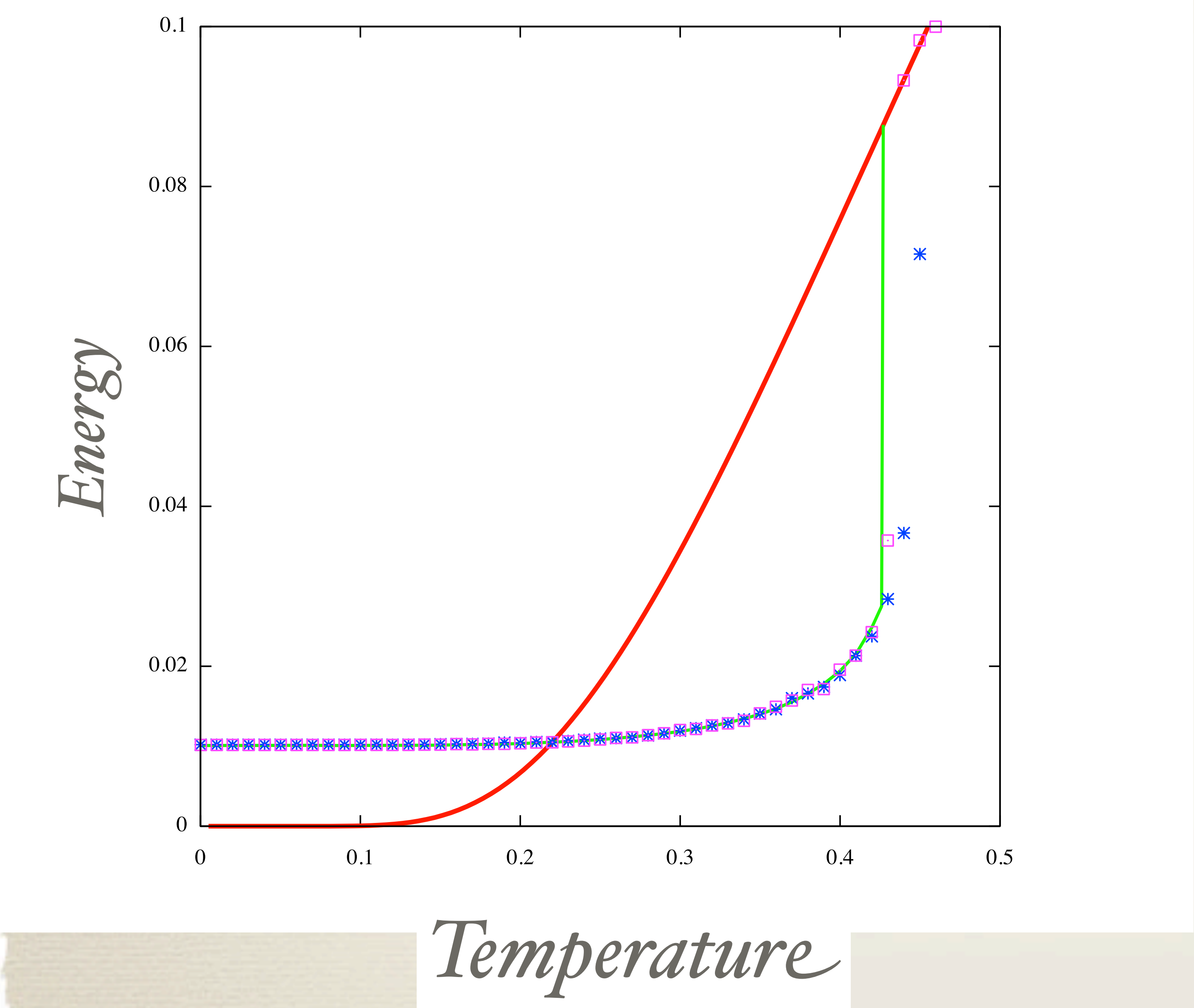

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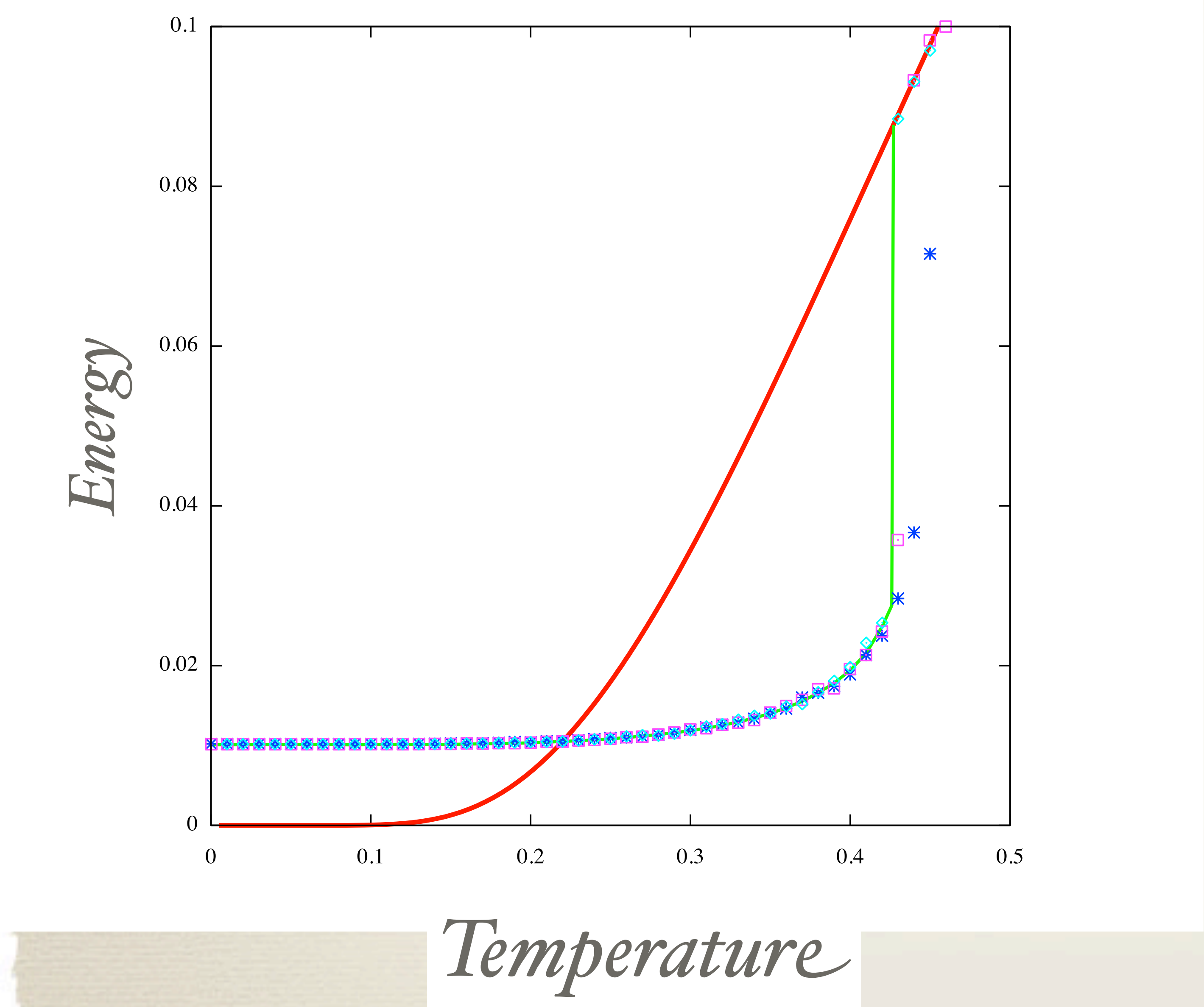

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Example 4:

Studying more complex Hamiltonians at low temperature

$$\mathcal{H}(\{S\}) = \sum_{ijk} \frac{1 + J_{ijk} S_i S_j S_k}{2}$$

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Studying more complex Hamiltonians at low temperature

$$\mathcal{H}(\{S\}) = \sum_{ijk} \frac{1 + J_{ijk} S_i S_j S_k}{2} \quad + \Gamma \mathcal{H}_{perturb}$$

Start with an equilibrated configuration at Γ=0 and increase Γ

Example 4:

Studying more complex Hamiltonians at low temperature

Example: *include a quantum transverse field!*

$$\mathcal{H}(\{S\}) = \sum_{ijk} \frac{1 + J_{ijk} S_i S_j S_k}{2} \quad \Rightarrow \quad \mathcal{H} = \sum_{ijk} \frac{1 + J_{ijk} s_i^z s_j^z s_k^z}{2} + \Gamma \sum_i s_i^x$$

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Studying more complex Hamiltonians at low temperature

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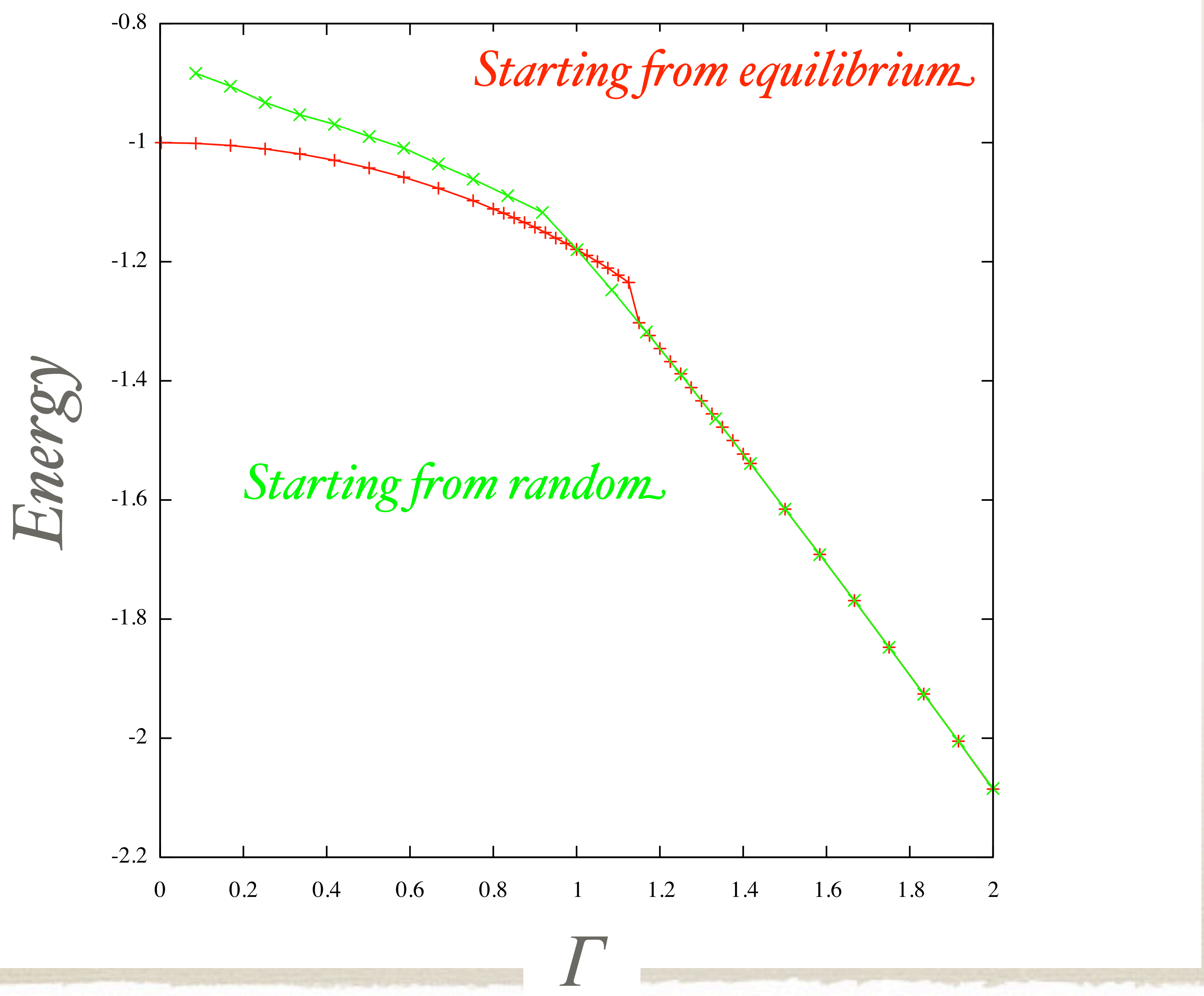

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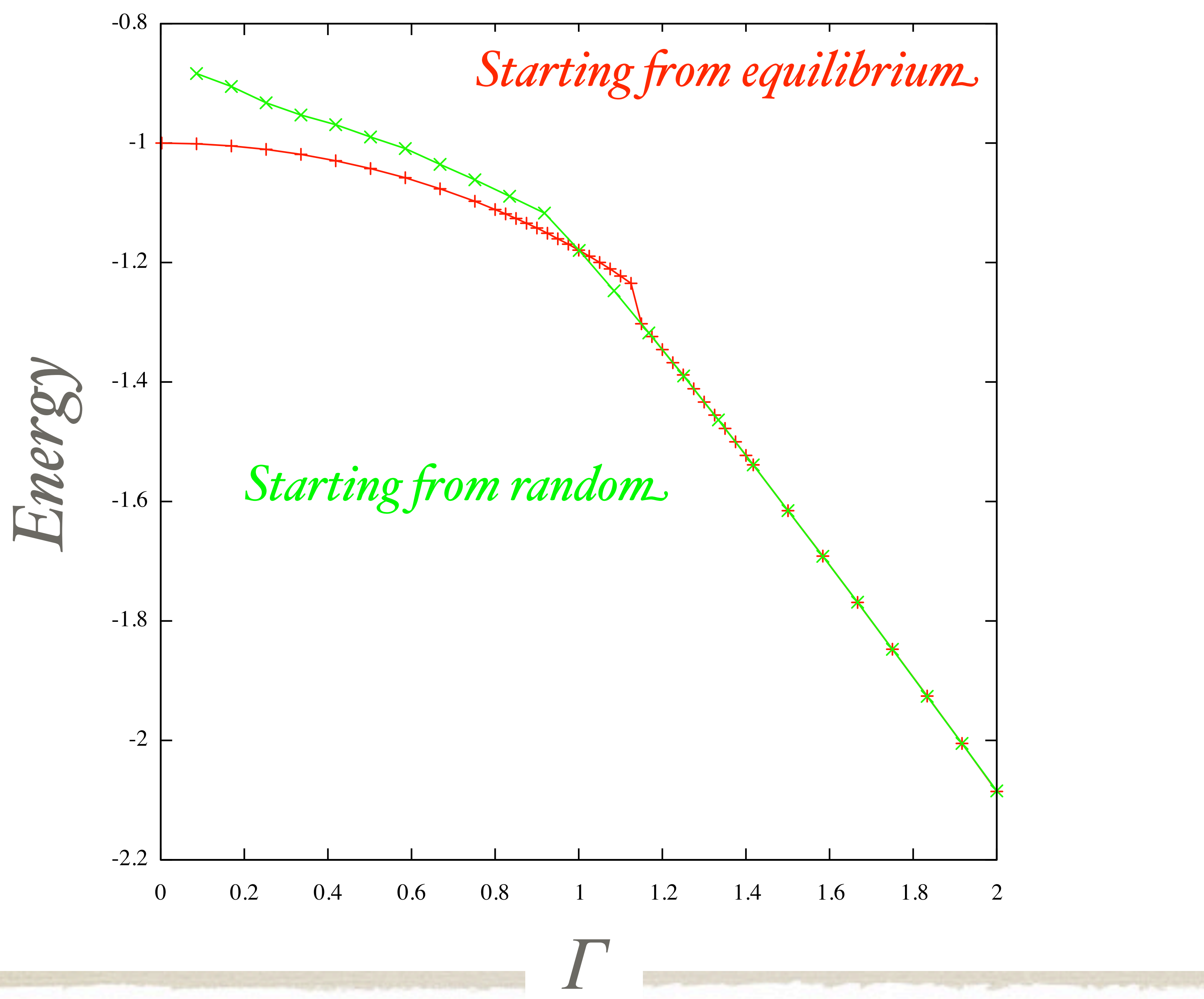

First order Quantum transition

Imply the failure of Quantum Annealing (or Quantum Adiabatic Algorithm)

Conclusions

A quiet planting is possible in many models.

- "Quiet" Planting does not change the properties of the ensemble up to the condensation threshold.
- Planted solutions are hard to find until the Kesten-Stigum threshold.
- Possibility to hide solutions (even a unique solution)

FK and L. Zdeborová:

* *Phys. Rev. Lett.* **102**, 238701 (2009)
* **arXiv:0902.4185, submitted in *SIAM Journal on Discrete Mathematics***
* **And more to come...**

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- Planted solutions are hard to find until the Kesten-Stigum threshold.
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There **is** a free lunch: instantaneous simulations.

- Many "mean field" models and random optimization models can be simulated efficiently using planting at zero or finite temperature.

FK and L. Zdeborová:

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